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Equalization of audio systems using Kautz filters with log-like frequency resolution

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ABSTRACT

This paper presents a new digital filtering approach to the equalization of audio systems such as loudspeaker and room responses. The equalization scheme utilizes a particular infinite impulse response (IIR) filter configuration called Kautz filters, which can be seen as generalizations of finite impulse response (FIR) filters and their warped counterparts. The desired frequency resolution allocation, in this case the logarithmic one, is attained by a chosen set of fixed pole positions that define the particular Kautz filter. The frequency resolution mapping is characterized by the allpass part of the Kautz filter, which is interpreted as a formal generalization of the warping concept. The second step in the actual equalizer design consists of assigning the Kautz filter tap-output weights, which is then in turn more or less a standard least-square configuration. The proposed method is demonstrated using measured loudspeaker and room responses.

1. INTRODUCTION

The utilization of digital signal processing (DSP) techniques for the equalization of audio systems, such as loudspeaker and room responses or combined loudspeaker-room responses, have been studied extensively for more than twenty years. A thorough list of references, an overview of methods, and considerations on known obstacles are provided by Mourjopoulos [15] [16]. Some additional references may also be cited [17, 13, 3, 7, 24, 8]. In particular, the problems associated with equalizer design by inverting an identified system function serve as the main motivation of this paper. The issue of non-minimum phase equalization can be considered as a fundamental question. On the other hand, it is also important to understand what a chosen equalizer configuration tries to accomplish from a broader point of view. The focus here is not particularly on non-minimum phase design considerations, but more likely on the undesired phenomenon that system function zeros tend to transform into problematic prominent and "long-ringing" resonances (poles) in an equalizer design by inverting the system function. Pre-processing, such as (complex) smoothing of the audio responses may be utilized in the equalizer design phase to overcome or reduce this effect [4]. An alternative is to identify the equalizer directly (in cascade with the system), for example, using least-squares techniques [14]. This latter approach is adopted in this paper in the context of Kautz filters, which can be seen to provide generalizations of FIR and warped FIR equalizer designs, respectively.

Kautz filters, or more suggestively, generalized transversal filters, are fixed-pole IIR filters that inherit many favorable properties of FIR filters, such as unconditional stability and robustness of design and implementation. In addition, the Kautz filter has a tapped transversal structure comprised of an allpass filter backbone and related tap-output allpole filters that together enforce the tap-output impulse responses to be mutually orthonormal for any choice of desired stable poles. The FIR filter is a Kautz filter with respect to the choice of poles at the origin.

We have demonstrated the potential applicability of Kautz filters for modeling such audio related measured responses as instrument body responses [18, 20, 22] and various room responses [19, 23], as well as loudspeaker equalizer design based on inverted minimum phase target responses [18, 21]. Here the objective is somewhat different: the desired overall allocation of frequency resolution of the equalizer is attained by choosing a fixed pole distribution that consists of complex conjugate pole pair with logarithmically spaced angles and a related mapping of the pole radii. The properties of such an equalizer are characterized in terms of the tap-output impulse responses and the allpass operator that defines the Kautz filter.

The Kautz filter tap-output weights are determined in the least-square sense from the corresponding normal equations: the Kautz filter tap-output responses (with respect to the response to be equalized as the input) are used to construct a correlation matrix and a correlation vector, respectively, which are then put into the form of a set of normal equations that is solved for the least-square optimal weights. The target signal (in forming the correlation vector) is the unit impulse signal, including a potential delay for the overall system. This approach also suggests the possibility of utilizing modified equalization targets,

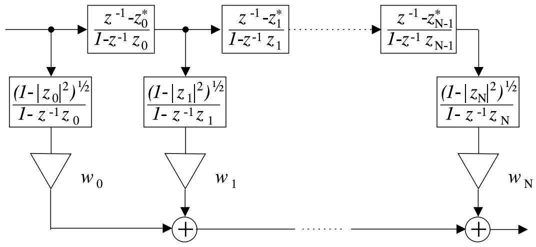

Fig. 1: The Kautz filter. For $z_i = 0$ in (1) it degenerates to an FIR filter, for $z_i = a, -1 < a < 1$, it is a Laguerre filter where the tap filters can be replaced by a common pre-filter.

as well as a generalized channel equalization scheme with respect to a generic input signal. It would also be possible (and interesting) to tackle the exact equalization task of a non-minimum phase response by optimizing with respect to different choices of Kautz equalizers and the required delay of the overall system. However, the purpose of this paper is to show that feasible approximative equalization is achieved with low-order Kautz equalizers.

2. KAUTZ FILTERS

The *Kautz filter* has established its name due to a rediscovery in the early signal processing literature [9, 1] of an even older mathematical concept related to rational representations and approximations of functions [25]. The generic form of a Kautz filter is given by the transfer function

$$\begin{aligned}\hat{H}(z) &= \sum\nolimits_{i=0}^{N} w_i G_i(z) \\ &= \sum_{i=0}^{N} w_i \left(\frac{\sqrt{1 - z_i z_i^*}}{1 - z_i z^{-1}} \prod_{j=0}^{i-1} \frac{z^{-1} - z_j^*}{1 - z_j z^{-1}} \right), \quad (1)\end{aligned}$$

where w_i, $i = 0, \ldots, N$, are somehow assigned tap-output weights. The orthonormal Kautz functions $G_i(z)$, $i = 0, \ldots, N$, are determined by any chosen set of stable poles: $\{z_j\}_{j=0}^{N}$, such that $|z_j| < 1$. The superscript $(*)$ denotes complex conjugation. Figure 1 is hopefully more instructive than formula (1).

Defined in this manner, Kautz filters are merely a class of fixed-pole IIR filters that are forced to produce orthonormal tap-output impulse responses. However, a Kautz filter is in fact more genuinely a generalization of the FIR filter and its warped counterparts, which will be demonstrated in terms of properties of the allpass filter that constitute the backbone of the tapped transversal structure of Figure 1.

The time-domain counterpart of (1), the Kautz filter impulse response, is given by

$$\hat{h}(n) = \sum_{i=0}^{N} w_i g_i(n), \tag{2}$$

where functions $\{g_i(n)\}_{i=0}^{N}$ are impulse responses or inverse z-transforms of functions $\{G_i(z)\}_{i=0}^{N}$. The meaning of orthonormality is specified most economically by defining the time-domain inner product of two (causal) signals $x(n)$ and $y(n)$,

$$(x, y) := \sum_{n=0}^{\infty} x(n) y^*(n). \tag{3}$$

Now, impulse responses $\{g_i(n)\}_{i=0}^{N}$ are orthogonal in the sense that $(g_i, g_k) = 0$ for $i \neq k$, and normal, since $(g_i, g_i) = 1$ for $i = 0, \ldots, N$.

A reasonable presumption in modeling a real response is that the poles should be real or occur in complex conjugate pairs. For complex conjugate poles, an equivalent *real Kautz filter* formulation [1], depicted in Fig. 2, prevents dealing with complex (internal) signals and filter weights. The allpass characteristics of the transversal blocks is restored by shifting the denominators in Fig. 2 one step to the right and by compensating for the change in the tap-output blocks. A mixture of structures in Fig. 1 and Fig. 2 is used in the case of both real and complex conjugate poles.

3. MODELING AND EQUALIZATION USING KAUTZ FILTERS

Our previous proposals for utilizing Kautz filters in relation to audio and acoustic systems have been based on approximative modeling of given (measured of somehow identified) target responses. The design procedure is then particularly straightforward: a desired pole set is generated with respect

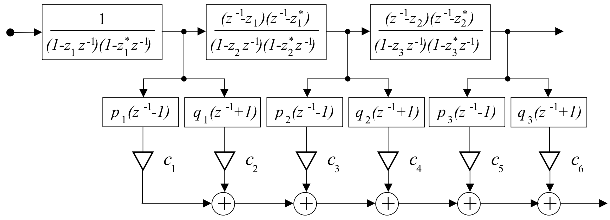

Fig. 2: One possible realization of a real Kautz filter, corresponding to a sequence of complex-conjugate pole pairs [1]. The normalization terms are $p_i = \sqrt{(1-\rho_i)(1+\rho_i-\gamma_i)/2}$ and $q_i = \sqrt{(1-\rho_i)(1+\rho_i+\gamma_i)/2}$, where $\gamma_i = -2RE\{z_i\}$ and $\rho_i = |z_i|^2$ are expanded polynomial coefficients of the second-order blocks.

to the target response $h(n)$ and the approximation (or the model) is composed as

$$\hat{h}(n) = \sum_{i=0}^{N} c_i g_i(n), \quad c_i = (h, g_i). \tag{4}$$

That is, the filter weights are the orthogonal expansion coefficients (Kautz-Fourier coefficients) of $h(n)$ with respect to the choice of basis functions. One of the favorable specialities of Kautz filter design compared to other IIR or pole-zero filter configurations is that the approximation is independent of rearrangement of the pole set, which implies means for reducing as well as extending the model by pruning, tuning, and appending poles, respectively. In addition, the use of orthogonal expansion coefficients correspond to least-square (LS) design with respect to the particular pole set, and as a consequence of the orthogonality, the approximation error (energy) is given simply as

$$\mathcal{E} = (h, h) - \sum_{i=0}^{N} c_i^2, \tag{5}$$

where (h, h) is the energy of the target response. As an alternative to the evaluation of $c_i = (h, g_i)$ using the inner product formula (3), the Kautz filter tap-output weights are also attained by feeding the signal $h(-n)$ to the Kautz filter and reading the tap outputs $x_i(n) = G_i[h(-n)]$ at $n = 0$: $c_i = x_i(0)$. That is, all inner products in (4) are implemented simultaneously using filtering. In the case of an FIR filter this would equal design by truncation of $h(n)$.

3.1. Least-square equalization using Kautz filters

As for any pole-zero model configuration, it is not in general a straightforward operation to invert or "flip" a Kautz model of a system into its inverse system or equalizing counterpart. In fact, this kind of direct inversion schemes are particularly unattractive from the point of view of Kautz filters because of the numerator configuration in the transfer function. The art of equalization by direct inversion techniques [17, 14, 3, 15] will not be considered further, but it is nevertheless interesting to notice that many of the consequential requirements posed upon an equalizer are inherently available in a Kautz filter configuration, namely,

- genuine IIR nature of the equalizer
- explicit control over the poles
- pure delays and allpass filters as building blocks
- linear in the weight-parameters, allowing a well-posed LS configuration

The equalization configuration that is more interesting from the Kautz filter point of view can be characterized as direct equalization: the equalizer (with impulse response $h_{eq}(n)$) is identified in cascade with the system $h(n)$ to be equalized, to meet or approximate the ideal equalization target

$$\hat{h}(n) = h(n) * h_{eq}(n) \approx \delta(n - \Delta), \tag{6}$$

where $\delta(\cdot)$ is the unit impulse, including a potential delay Δ. Also in this direct configuration it is still possible to use an inverted target response as the basis of design, directly as in (4) or for example to identify just the poles. These alternatives will be demonstrated in the loudspeaker equalization example. It is noteworthy that some of the known problems associated to using an inverted response to construct the equalizer, such as "ringing poles" and truncation effects, do not (necessarily) pass on to the Kautz equalizer. This may by itself reduce the need for complicated pre-processing operations. In addition, even this simple approximation setup is by no means constrained to minimum-phase or magnitude-only equalization: the target response may obviously be constructed to take into account phase as well as magnitude equalization.

A more genuine form of direct equalization is provided by the least-square configuration [14]: the square error in the approximation (6) is minimized with respect to the equalizer parameters (filter coefficients). In terms of the Kautz equalizer, the tap-output weights $\{w_i\}$ are optimized according to

$$\min_{w_i} \left(\sum_n (\hat{h}(n) - \delta(n - \Delta))^2 \right), \tag{7}$$

where the equalizer response

$$\hat{h}(n) = \sum_{i=0}^{N} w_i x_i(n), \quad x_i(n) = g_i(n) * h(n), \tag{8}$$

is the Kautz filter response to the input $h(n)$. Using system identification terminology, the equalization setup is an output-error configuration with respect to a special choice of model structure. It can even be considered as a generalized linear prediction formulation. Furthermore, it is a quadratic LS problem with a well-defined and unique solution that is obtained from the corresponding normal equations: if the Kautz equalizer tap-output responses $x_i(n) = g_i(n) * h(n)$ are assembled into a "generalized channel convolution matrix"

$$\mathbf{S} = \begin{pmatrix} x_0(0) & \cdots & x_N(0) \\ x_0(1) & \cdots & x_N(1) \\ \vdots & \ddots & \vdots \\ x_0(L) & \cdots & x_N(L) \end{pmatrix}, \tag{9}$$

then the normal equations submit to the matrix form

$$\mathbf{S}^T\mathbf{S}\mathbf{w} = \mathbf{s}, \quad \mathbf{w} = [w_0 \ \cdots \ w_N]^T, \tag{10}$$

where $\mathbf{s}$ is the (cross-)correlation vector between the tap-output responses and the desired response $d(n)$, $s_i = (d, x_i)$. The matrix product $\mathbf{S}^T\mathbf{S}$, where $(\cdot)^T$ denotes transpose of a matrix, implements correlation analysis of the tap-output responses, (x_i, x_j), in term of the inner product (3), where it is presumed that the Kautz filter responses are real-valued. In the case of an impulse as the desired response, $d(n) = \delta(n - \Delta)$, the correlation vector simply picks the $(\Delta + 1)$th row of the matrix $\mathbf{S}$,

$$\mathbf{s} = [x_0(\Delta) \ \cdots \ x_N(\Delta)]^T. \tag{11}$$

The solution of the matrix equation (10) is

$$\mathbf{w} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{s} \quad (12)$$

and it provides the LS optimal equalizer tap-output weights with respect to the choice of Kautz functions.

There are obviously practical as well as more principled aspects that still have to be specified in the proposed Kautz equalizer configuration. For example, some kind of compensation of the low- and high-frequency roll-offs of the response to be equalized is usually needed in the design phase to attain desired equalization profiles. The subject of coefficient (and related signal) scaling should also be taken into account. However, these considerations do not differ too much from the ones encountered already in the FIR equalizer case. A somewhat more specialized question is the choice of the "correlation length" L. Our choice is to use a sufficiently large $L > M$, where M is the length of the response $h(n)$, that in practice drains out the memory of the Kautz equalizer for the input $h(n)$. For a particular choice of a Kautz filter this length could also be quantized since the Kautz filter response is a superposition of decaying exponential components. This is in fact not a big issue due to the nature of the configuration, and in practice any $L > M$ will collect the essential part of the "correlation energy", for example, the choice $L = M + N$ as in the conventional LS setting.

3.2. Log-scale modeling using Kautz filters: pole distributions and resolution descriptions

The choice of poles has not been considered so far. In the case of modeling a given target response, the poles may be optimized using an iterative method [18] that resembles the denominator part of the well-known *Steiglitz-McBride method*. This pole optimization process, entitled the *BU-method*, will be utilized in the loudspeaker equalization Case 1. However, in an identification setup, such as the LS equalization configuration, there are really not too convincing methods available. A gradient based search method has been proposed in the case of an conventional IIR equalizer [12]. The Kautz filter can be seen as transitional form between strict pole-zero modeling and an FIR-like configuration using a "slightly recursive" structure. This property is utilized in the following by incorporating desired overall frequency resolution allocation by the choice of pole distributions.

A well-known way to attain flexibility in the frequency resolution description is to use warped filter configurations [5]. Warped structures may also be used to approximate logarithmic allocation of frequency resolution, but the approximation is not particularly good. Parallel allpass structures have been proposed to attain more accurate descriptions on a logarithmic scale [6]. In this paper the desired log-like frequency resolution is produced simply by choosing the Kautz filter poles according to a logarithmically spaced pole distribution. In polar coordinates, a set of poles

$$\{z_1, \ldots, z_N\} \rightleftharpoons \{r_1 e^{j\omega_1}, \ldots, r_N e^{j\omega_N}\} \quad (13)$$

is generated, where the angles $\{\omega_1, \ldots, \omega_N\}$ correspond to a logarithmic spacing for a chosen number of points in the interval $[\omega_1 \ \omega_N] \subset]0 \ \pi[$. We choose the corresponding pole radius as an exponentially decreasing sequence

$$r_i = \alpha^{\omega_i}, \ \alpha = e^{\ln(r_1)/w_1}, \ r_1 < 1. \quad (14)$$

This choice of pole radii will provide an approximative constant-Q resolution for the Kautz equalizer. Each pole is then "duplicated" with its complex conjugate to produce a real Kautz filter (Fig. 2). From a more practical point of view, the poles are generated using the formulas

$$\omega_i = \quad 2\pi f_i / f_\mathrm{s} \quad (15)$$

$$p_i = \quad R^{\omega_i/\pi} e^{\pm j\omega_i} \quad (16)$$

where p_i is the i^{th} pole pair $\{z_i, z_i^*\}$, f_i is the corresponding frequency (in Hz), and R is the pole radius corresponding to the Nyquist frequency $f_\mathrm{s}/2$, where f_s is the sample rate (in Hz).

Figure 3 gives an example of the magnitude spectrum of Kautz filter tap-output impulse responses for the choice of logarithmically spaced poles. Each "resonance" is represented by a pair of responses according to the dual output related to the corresponding pais of poles (Fig. 2). The pole set that generated the response is depicted in Fig. 4(a). Figures 4(b)–(d) characterize the attained frequency resolution allocation from the perspective of the transversal allpass part of the Kautz filter. The relationship

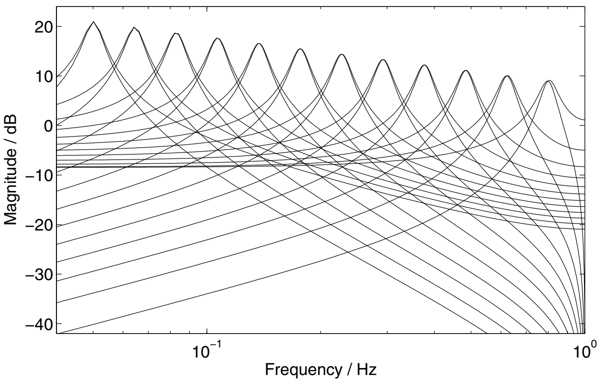

Fig. 3: Magnitude responses of the Kautz filter tap-output impulse responses with respect to the proposed logarithmic distribution of poles.

between the allpass operator and the corresponding orthonormal filter structure (the Kautz filter) is explained more thoroughly in [21]. The negated phase function of the allpass filter (solid line in Fig. 4(c)) can be interpreted as the frequency scale mapping, whereas its derivative in Fig. 4(d) characterizes the frequency resolution allocation introduced by the choice of poles. The upper curve in Fig. 4(d) is the sum of magnitude responses of Fig. 3, which shows quite explicitly that the chosen resolution description is somehow meaningful.

The effect of pole radius tuning is demonstrated in Fig. 5 using various displaying scales for the allpass phase mappings and corresponding derivatives. It is noteworthy that the phase responses are quite insensitive to relatively big changes in the radius profile. The pole distributions are chosen purposefully close to the unit circle to produce spiking in the resolution mapping: there is a clear transition from a localized resolution to a smoother overall description when the radius parameter R in Eq. (16) is reduced. Another notable aspect is that the smoothness scales well (evenly for the whole frequency range) with the chosen rule for the pole radius mapping.

4. EXAMPLES OF LS EQUALIZATION USING KAUTZ FILTERS

In this section we apply the proposed principles of Kautz filter design for the equalization of loudspeaker and room responses. The examination of

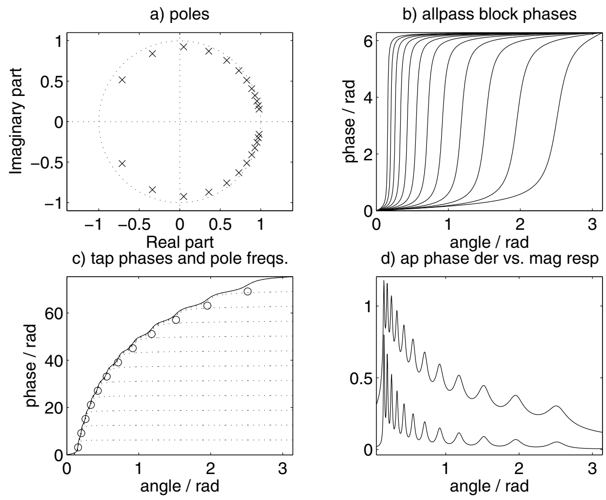

Fig. 4: a) The poles set, b) negated phase functions of 2nd order allpass blocks, c) (allpass) tap-output phases – overall phase (solid line) is the frequency scale mapping (circles indicate pole positions on this mapping), and d) phase derivative (lower curve) w.r.t. sum of Kautz filter tap-output magnitude responses (scaled).

many practical issues would need a more thorough investigation; here we take only some cases that illustrate the characteristics and capabilities of Kautz equalizers.

4.1. Loudspeaker equalization, Case 1

In this first example of Kautz filter equalization, an inverted minimum-phase target response (with respect to a measured loudspeaker response) is used both directly and indirectly to construct the equalizer. The Kautz filter poles are generated in both cases using a warped counterpart of the BU-method [20] with respect to the inverted target response. The equalizer filter order is chosen to be 38 (18 complex conjugate pole pairs and two real poles). The purpose of this example is to demonstrate that two very different equalizer parametrization schemes, corresponding to Equations (4) and (8), respectively, produce very similar magnitude response equalization results, as depicted in Fig. 6.

The early part of the measured loudspeaker impulse response and the LS equalized response are displayed

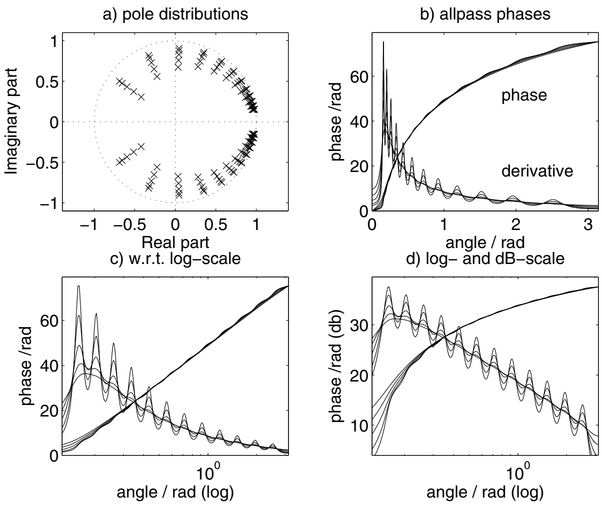

Fig. 5: Allpass filter characteristics for varying pole radius damping, a) pole sets, b) phase functions and phase derivatives, c) on log-scale, and d) in dBs on log-scale.

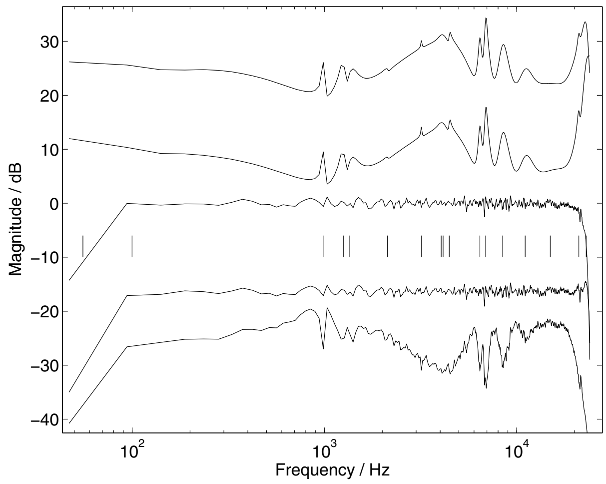

Fig. 6: Magnitude responses from bottom to top: measured loudspeaker response, LS equalized (method (8)), pole frequencies, equalized using (4) w.r.t. inverted target, corresponding equalizer responses.

in panels (a) and (b) of Fig. 7. In Fig. 7(c) the LS equalizer is designed with respect to a delay $\Delta = 12$ in the target of equalization. The pole set that is generated from a minimum phase target response is not very good at producing pure delay components, which results also in inefficiency in magnitude equalization (not shown). A somewhat trivial way to attain better equalization is to include zeros in the Kautz filter pole set: in Fig. 7(d) the equalizer is equipped with 12 additional poles at the origin, that is, part of the Kautz filter is implemented as an FIR filter substructure.

4.2. Loudspeaker equalization, Case 2

Another example of Kautz equalizer design is presented in Fig. 8. It depicts the same loudspeaker response as in Case 1 having a relatively non-flat magnitude response (curve (a)). The response is corrected by a 24^{th} order (12 pole pairs) Kautz filter with logaritmically positioned pole frequencies between 80 Hz and 23 kHz (indicated by vertical lines in the middle of the figure) and $R = 0.03$ (see Eq. 16). After low- and high-frequency roll-off compensations to avoid boosting off-bands of the speaker, as shown by curve (c), the equalizer filter resulting from Kautz LS equalization shows its magnitude response in curve (d). The equalized response is plotted in curve (e) and as a 1/3-octave smoothed version in curve (f).

Filter orders from 8 up (4 pole pairs) give useful results in this case, although the selection of order and pole positions may introduce considerable variation in flatness of the result. Therefore full optimization requires a search over sets of poles and filter orders, in spite of the fact that the LS procedure itself always gives optimal tap coefficients for a given fixed order and pole set.

Curve (g) in Fig. 8 demonstrates the effect of Kautz pole radii selection. In this case the poles are set too close to the unit circle ($R = 0.8$), thus the frequency ranges around pole frequencies get too much emphasis. Otherwise, in most cases, the selection of pole radii is not critical at all. Even very small radii, such as $R = 10^{-5}$, work well in this case.

Comparison of curves (e) and (g) explains also clearly why LS equalization using the Kautz filter configuration behaves favorably with zeros in the response to be equalized, while exact inversion of a response with deep dips results in undesirable peaks and long-ringing decay times in the equalizer [4]. In

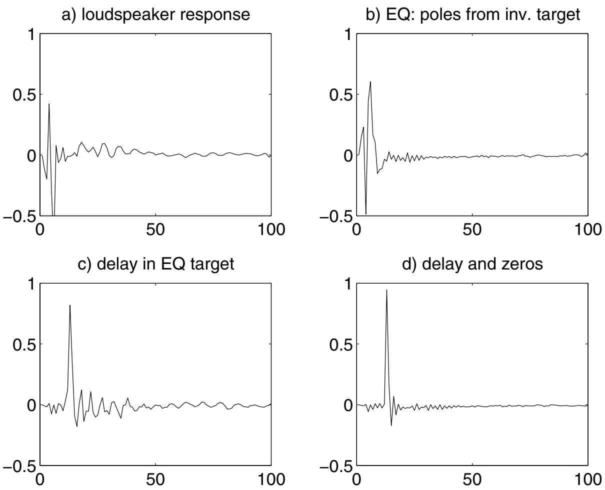

Fig. 7: Early part of time-domain responses: a) measured loudspeaker, b) LS equalized (Kautz filter order 38), c) using the same set of poles and including delay ($\Delta = 12$) in target, and d) by including 12 poles at the origin and delay ($\Delta = 12$) in target.

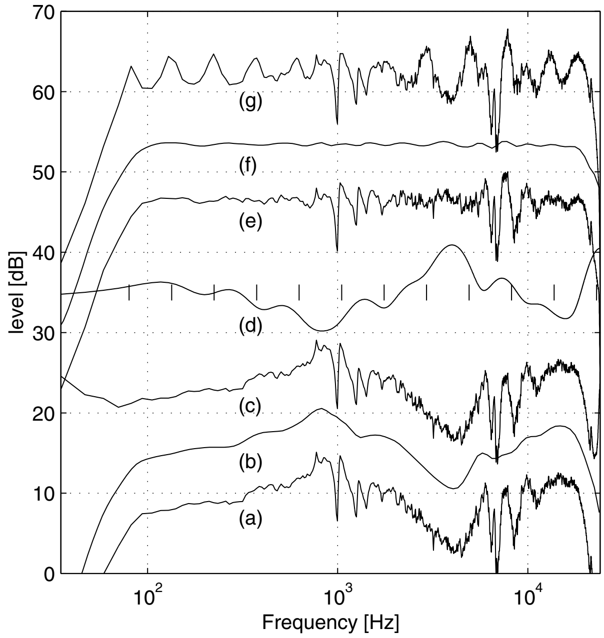

Fig. 8: Basic example of loudspeaker response equalization by Kautz filter inverse modeling. From bottom up: (a) measured magnitude response, (b) same one 1/3-octave smoothed, (c) low and high roll-off compensation, (d) magnitude response of 24$^{\text{th}}$ order (12 pole pairs) Kautz equalizer, (e) equalized magnitude response, (f) same one 1/3-octave smoothed, and (g) Kautz equalized response with $R = 0.03$. Vertical lines at 35 dB level indicate the frequency positions corresponding to logarithmically spaced pole angles.

Kautz filters the pole radii determine the maximum Q values of resonances. If the pole radii are selected conservatively, no excess peaking and ringing of resonances appear in the equalizer response.

4.3. Loudspeaker equalization, Case 3

In this subsection we demonstrate further the use of Kautz modeling in loudspeaker equalization including both magnitude and phase correction. While in previous studies with warped and Kautz filters [7, 5, 21] we have applied magnitude-only equalization, it is interesting to investigate the ability of Kautz inverse models to correct also the phase behavior.

For clarity of phase curves, it is preferable to demonstrate the phase correction by using a synthetic (simulated) loudspeaker response instead of a real measured one. Figures 9 and 10 depict the magnitude and group delay behaviors of an idealized two-way loudspeaker. It consists of a low-frequency driver in a wented box (4$^{\text{th}}$ order highpass at 80 Hz) and a high-frequency driver (up to 18 kHz), both with flat response between roll-off frequencies. They are combined with a second order Linkwitz-Riley crossover network [10, 11], which in an ideal case results in a flat magnitude response at the main axis.

In this particular case we investigate a loudspeaker where the acoustic center of the high-frequency driver is 17 cm behind the acoustic center of the low-frequency unit. This means a temporal non-alignment of about 0.5 ms, which results in ripple of the main axis magnitude response (‘Original’ in Fig. 9) and similarly a non-flat group delay response (‘Original’ in Fig. 10). The magnitude response error of this amount is easily audible. Although the group delay deviation remains within 1 ms above 300 Hz, which is hardly noticeable in practice, it is interesting to check how the phase correction by Kautz

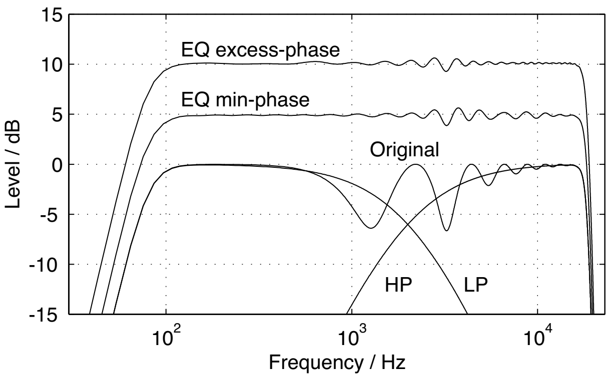

Fig. 9: Magnitude responses of the simulated loudspeaker: LP = low-pass crossover; HP = high-pass crossover; original = response due to driver distance misalignment; minimum-phase equalized; excess-phase equalized response.

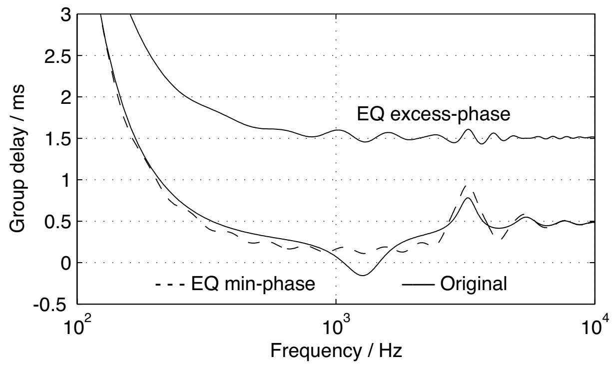

Fig. 10: Group delay responses of the simulated loudspeaker: original = ripple due to driver distance misalignment; minimum-phase equalized; excess-phase equalized response with extra group delay.

LS equalization works. This brings necessarily increased latency beyond the maximum group delay of the original response.

Curves 'EQ min-phase' in Figs. 9 and 10 show the magnitude and group delay responses of the simulated loudspeaker when a Kautz equalizer is designed based on the minimum-phase part of the loudspeaker impulse response. A Kautz filter of 18 pole pairs was designed with logarithmic distribution of poles between 80 Hz and 23 kHz and pole radius coefficient $R = 0.1$. High- and low-frequency roll-offs are compensated in EQ design to remain as they were originally. After equalization the magnitude response is flat within ± 1 dB, while the group delay (dashed line) is not essentially improved.

Curves 'EQ excess-phase' in Figs. 9 and 10 illustrate the results of magnitude plus phase equalization with a Kautz LS equalizer. In this case the target response of the equalized system is given as a delayed impulse, with a latency higher than the maximum delay of the loudspeaker itself. The target group delay was set here to 1.5 ms (66 samples at 44.1 kHz sample rate). A Kautz equalizer was designed with 8 logarithmically distributed pole pairs within 80 Hz to 23 kHz, with $R = 0.05$, plus 96 poles at the origin. Notice that the latter ones correspond again to FIR filter behavior, so that the equalizer is a mixture of an FIR and an IIR filter.

After applying excess-phase equalization the magnitude response in Fig. 9 is again within ± 1 dB, while the group delay curve in Fig. 10 has ripple less than ± 0.1 ms. (The growth of low-frequency group delay comes from the high-pass behavior of the loudspeaker, which is not compensated for.)

4.4. Room response equalization, Case 4

In this subsection we examine a basic example of loudspeaker plus room response correction using Kautz LS equalization. The loudspeaker used had a lower roll-off frequency of about 80 Hz, compensated in target response design. The room was a listening room of 33 m^2 with fairly well controlled acoustics. Figure 11 shows the first 5 ms of the measured impulse response in pane (a) and magnitude response in pane (c) in full resolution and 1/3-octave smoothed.

A Kautz equalizer of order 24 (12 pole pairs) was designed with logarithmically positioned pole frequencies between 50 Hz and 20 kHz, using the pole radius parameter value $R = 0.5$. The resulting impulse response and magnitude response are plotted in Fig. 11, panes (b) and (d). The magnitude response is flattened as desired. In the impulse response some low-frequency oscillation is damped, but the peaks corresponding to reflections from surfaces cannot of course be canceled out by such a low-order equalizer. However, even equalizer orders of

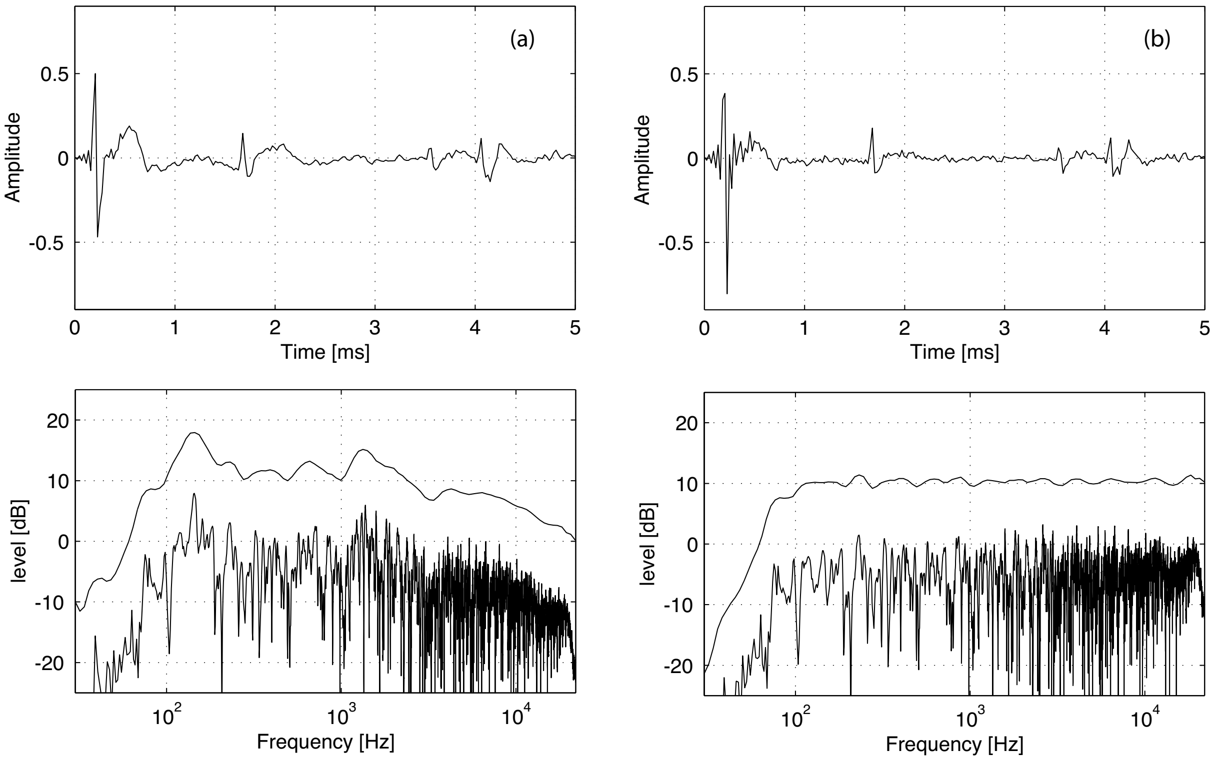

Fig. 11: Kautz equalization of a room response: (a) first 5 ms of the impulse response, (b) first 5 ms of the equalized impulse response, (c) original magnitude response, and (d) Kautz equalized magnitude response. In magnitude responses the lifted upper curves are computed through 1/3-octave smoothing.

8–12 (4–6 pole pairs) seen to provide useable equalization results in this particular case.

In principle the Kautz equalization is capable to approach perfect inversion or any resolution of magnitude and phase equalization. In practice there are however limitations, such as ability to do major corrections in phase behavior, e.g., proper temporal envelope manipulation as done in [4]. This remains a topic for future research.

5. DISCUSSION AND CONCLUSIONS

Many DSP techniques have been proposed earlier for the equalization of audio reproduction channels, either for loudspeaker only or for combined loudspeaker and room responses. We have earlier investigated different frequency warping techniques, including Kautz filters, which allow for maximum flexibility of frequency resolution control in equalizer design [21]. The previous Kautz filter equalizers were designed as models for responses that were separately inverted for given reproduction system responses. For simplicity, the inversion was done only for the minimum-phase part.

In the present study we have extended the use of Kautz filter equalizers to more general cases by systematic design of inverse filter models directly from given impulse responses of audio reproduction systems. The novelty is to apply least square optimal design of Kautz filter tap coefficients for a fixed set of system poles. Logarithmic frequency resolution is approximated by setting the pole angle distribution logarithmically and by controlling the pole radii to approximate constant-Q behavior. This is conceptually similar to inverse (FIR) filter design through linear prediction for uniform frequency resolution.

An advantage of the new method is that it can be applied to non-minimum phase EQ design, as demonstrated for loudspeaker equalization. Another favorable feature is the inherent control of sharp magnitude response dips to avoid sharp peaks and long-ringing temporal decays in the equalizer design. This is done simply by limiting the Q-values of the Kautz system pole pairs.

After introducing Kautz filters in Section 2 and the new principle of Kautz equalization in Section 3, a set of case studies are presented in Section 4. These include different loudspeaker response corrections, including non-minimum phase equalization. A typical listening room case is also studied for combined loudspeaker and room response equalization. This is primarily magnitude-only equalization; more detailed control of phase behavior is left to future research.

Low-order Kautz equalizers (4 to 12 pole pairs) are found in practice to yield good magnitude response correction. Phase correction may need higher orders with a set of system poles at the origin and excess delay required for the target response.

Kautz filters are know to be numerically robust due to the allpass type of elements used in the implementation. For low orders it is possible to map the transfer function to direct form IIR structures for maximal efficiency on DSP processors. This and many other practical questions remain however out of the scope of this paper, which is the first step in introducing this new concept of audio equalizer design.

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