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Sound field reproduction with stochastic secondary sources

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ABSTRACT

Sound field reproduction (including wavefield synthesis and spectral division method) is a state-of-the-art technique, aiming to physically reproduce an arbitrary sound field, usually generated by a virtual sound source. To achieve this, densely spaced loudspeaker array, termed as secondary source distribution is driven by a driving function derived either in spatial or spectral domain.

The synthesis is usually modelled as reproduction, applying continuous point source distribution. However, practial realization applies extended vibrating surfaces (e.g dynamic loudspeaker), exhibiting stochastic properties. In this contribution we give a treatise on the synthesis applying extended secondary sources. Stochastic behaviour is modelled as additive noise on source extension function and on the driving function. The proposed model can incorporate the effect of stochastic speaker sensitivity, surface rugosity, mechanical anisotropy and stochastic modal behaviour.

It is investigated, how the different stochastic properties contribute to the radiated sound field. Based on the sensitivity of human auditory system it is examined, how the stochastic behaviour of the secondary sources influences the localization of the synthesized virtual sound source. Besides analytical examination the results of Monte Carlo simulations are presented.

1 INTRODUCTION

Sound reproduction methods, applying a large number of densely spaced loudspeakers have been the target of research for over a decade. The aim of these techniques is to physically synthesize a desired sound field over an extended listening area. Well known approaches in this context are wave field synthesis (WFS) [1, 2, 3], spectral division method (SDM) [4, 5] and higher-order Ambisonics (HOA) besides numerous numerical approaches.

Traditional techniques, like WFS and SDM involve identical acoustical point sources along an infinite line as secondary source distribution for the reproduction of the desired sound field. The general theory of spectral division method gives us the possibility to derive an analytical formulation to the case where directional secondary sound sources are employed, as long as we suggest identical secondary sources [6]. Secondary source directivity can be adapted also to the theory of wave field synthesis within the limits of the stationary phase approximation, still, assuming identical source elements [7]. However,

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the requirement of applying linear, continuous distribution of identical point sources is barely fulfilled in practice. Typically sound field reproduction is realized by using electrodynamic or electrostatic loudspeakers arranged along a line, thus realizing distribution of acoustical monopoles or dipoles.

Obviously, real-life loudspeakers – even from the same type – are not identical; both their temporal and spatial characteristics are different. These random differences can be treated as stochastic transducer characteristics. To the author's knowledge the effects of the stochastic properties have not been investigated in details yet. The objective of the research is to give a rigorous mathematical description of the radiated noise and to examine if these stochastic artifacts may have an audible effect on the synthesized sound field.

In the present paper after a short overview on the theoretical basis of sound field reproduction we present a synthesis model, using deterministic, then stochastic extended secondary source elements. Two main stochastic components are taken into consideration: stochastic loudspeaker sensitivity originating from stochastic electric and electrodynamic parameters of the transducer, and mechanical noise representing stochastic mechanical material properties. These latter effects are also investigated using mechanical finite element simulations carried out for a typical electrodynamic loudspeaker model. It is investigated, how these two noise components influence the synthesized sound field of a virtual point source.

2 THEORETICAL OVERVIEW

2.1 Theory of sound field reproduction

In this section a short overview is given on the theory of sound field reconstruction, focusing mainly on the wave field synthesis, as in the latter investigation the traditional wave field synthesis driving functions are used. However, as it was pointed out in the author's previous work by applying farfield-approximations to spectral division method, the SDM and WFS can be treated equivalent [8].

The general problem arrangement can be seen in figure 1. In this configuration the generated wave field can be written as the sum of the wave fields of individual monopoles, called *secondary sources*, that form the *secondary source distribution*. Our aim is to find the secondary sources' *driving function* that results in a generated field equal to that of the *virtual source*. The resulting sound field, generated by the secondary source distribution on the line $[x\ 0\ 0]^{\mathrm{T}}$ can be written as a convolution form:

$$P(x, y, \omega) = \int_{-\infty}^{\infty} Q(x_0, \omega) G(x - x_0, y, \omega) \mathrm{d}x_0, \tag{1}$$

where $Q(x, \omega)$ is the driving function, while $G(x, y, \omega)$ is the three-dimensional free-field Green's function [9]. In the $z = 0$ plane it is given by

$$G(x, y, \omega) = \frac{1}{4\pi} \frac{\mathrm{e}^{-\mathrm{j}k\sqrt{x^2+y^2}}}{\sqrt{x^2 + y^2}}. \tag{2}$$

For the sake of brevity the notation of frequency dependence will be further omitted.

The mathematical basis of traditional WFS is the Rayleigh I integral, a special case of Kirchhoff-Helmholtz integral formula [10, 11]. The Rayleigh integral describes the wave field of an infinite plane radiating to the infinite halfspace, so the Sommerfeld radiation condition is fulfilled. In the aspect of WFS, the Rayleigh I integral states that the pressure field of a *virtual source* behind an infinite plane can be synthesized in front of the plane with a planar monopole distribution, driven with two times the normal velocity component created by the virtual source on the plane.

Practical implementations of wave field synthesis employ loudspeakers located along a horizontal line instead of a plane. This is termed as 2.5-dimensional synthesis. In this configuration perfect reproduction is only possible on the *reference line* that is parallel to the secondary source line. In other locations of the *synthesis plane* – a horizontal plane containing the secondary source line – amplitude

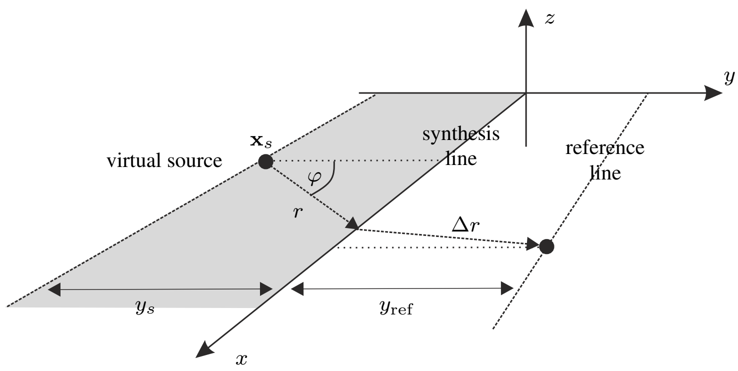


Figure 1: Geometry for the derivation of 2.5-dimensional synthesis operator in spatial and wave number domain

errors occur. Traditional WFS 2.5-dimensional driving function formulation was given by e.g. Verheijen, applying the stationary phase approximation to the Rayleigh I integral [1]. With the notation shown in figure 1 and for a virtual monopole at $\mathbf{x}_{\mathrm{s}} = [x_s \; (-y_s) \; 0]^{\mathrm{T}}$ the driving function takes the form

$$Q_{\mathrm{WFS}}(x) = \sqrt{\frac{\mathrm{j}k}{2\pi}} \sqrt{\frac{y_{\mathrm{ref}} y_s^2}{y_{\mathrm{ref}} + y_{\mathrm{s}}}} \frac{\mathrm{e}^{-\mathrm{j}kr}}{r^{\frac{3}{2}}}. \tag{3}$$

For the sake of simplicity the origin of the coordinate system is chosen so that the virtual monopole is located at $x_s = 0$.

As it was pointed out in [4], the convolution form of the Rayleigh integral (1) can be transformed into a spectral multiplication, thus the spectrum of the driving function can be written as

$$\tilde{Q}_{\mathrm{SDM}}(k_x) = \frac{\tilde{P}(k_x, y_{\mathrm{ref}})}{\tilde{G}(k_x, y_{\mathrm{ref}})}, \tag{4}$$

where $\tilde{P}(k_x, y_{\mathrm{ref}})$ and $\tilde{G}(k_x, y_{\mathrm{ref}})$ are the spectra of the sound field of the virtual source and the Green's function at the origin, measured on the reference line. This is the concept of the SDM method.

2.2 Modelling Extended Sources

In this treatise a similar treatment of extended sound sources is used as it was given by [1]. We assume that the extended source – the loudspeaker diaphragm – is located at the $y = 0$ plane, vibrating with a normal surface velocity $V_{\mathrm{n}}(x, 0, z)$. The radiated field in the plane $z = 0$ can be written analytically utilizing the Rayleigh I integral theorem, again in a convolutional form:

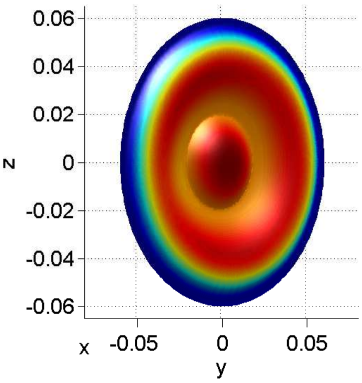
$$P(x, y, 0) = -\frac{\mathrm{j}\rho_0 ck}{2\pi} V_{\mathrm{n}}(x, 0, z) * *G(x, y, 0) \tag{5}$$

If the vertical size of the speaker is small compared to the wavelength, then the kernel of the convolution (the Green's function) can be made independent of the z-dimension and the radiated sound field can be written as a one-dimensional convolution along the x-axis:

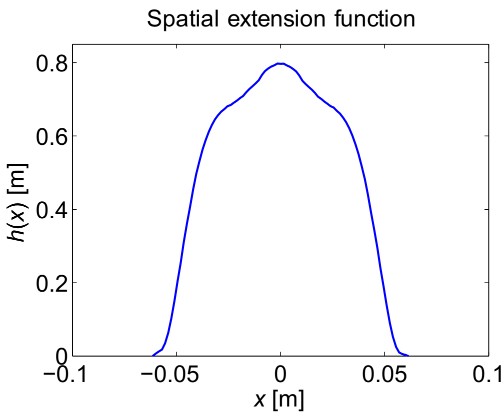
$$P(x, y, 0) \approx h(x) * G(x, y, 0), \tag{6}$$

where $h(x)$ is a low-frequency farfield approximation of the surface velocity function:

$$h(x) = -\frac{\mathrm{j}\rho_0 ck}{2\pi} \int_{-\infty}^{\infty} V_n(x, 0, z) \mathrm{d}z. \tag{7}$$



(a)



(b)

Figure 2: (a) Amplitude of excursion of the FEM loudspeaker model and (b) spatial extension function, calculated numerically at $1\ \mathrm{kHz}$

From here the one-dimensional function $h(x)$ will be termed as *spatial extension function* or strength function. Numerical simulations showed that this approximation holds well to the frequency range of 2-3 kHz besides about the diaphragm diameter of 0.1-0.2 m.

Using the approximation and by utilizing the associative property of convolution, the reconstructed sound field of a virtual monopole can be written as

$$P(x, y) = Q(x) * h(x) * G(x, y, 0), \tag{8}$$

where $Q(x)$ is the reconstruction driving function, derived either by WFS or SDM technique.

Note, that the discretization of the secondary source distribution is omitted here. This means that the synthesis could be interpreted physically as synthesis using overlapping loudspeaker distribution. Naturally it is not a realizable condition, yet it makes the mathematical description less complicated. The application of non-overlapping loudspeaker array could be treated analytically by sampling the driving function $Q(x)$ before the convolution with the loudspeaker extension function.

To examine how the extension function varies as the function of diaphragm geometry and frequency a mechanical finite element model was created. Simulation was carried out for a simple loudspeaker model with a polypropylene diaphragm, with the diameter of 13 cm – which is a feasible size in the aspect of wave field synthesis – driven with the force exerted by the voice coil. The membrane parameters, material properties and dimensions were taken from related research [12].

The simulation results were used to approximate the extension function $h(x)$ by evaluating equation (7) at 1 kHz. The resulting function is shown in figure 2.

3 MODEL FOR STOCHASTIC EXTENDED SOURCES

So far we considered sound sources with deterministic properties. Obviously, real-life speakers always have slightly different characteristics, even if they are of the same type. The main objective of the present work is to investigate how these stochastic properties influence the radiated sound field described by equation (8). In the present section we take two stochastic components into consideration:

- *Stochastic sensitivity* models that each loudspeaker has slightly different electromechanical transducer gain, depending on electrical, magnetic and mechanical properties. The sensitivity noise can be modeled as an additive noise $n_{\mathrm{s}}(x)$ to the nominal sensitivity. If the sensitivity is normalized, then the resulting noisy driving function can be written as:

$$Q_n(x) = Q(x)(1 + n_{\mathrm{s}}(x)) = Q(x) + Q(x)n_{\mathrm{s}}(x), \tag{9}$$

The result shows that sensitivity noise acts as a multiplicative noise on the driving function.

- *Mechanical anisotropy* is introduced to model the stochastic effects originating from surface rugosity, space dependent material properties, such as material density or stiffness. As these components will directly affect the surface velocity function, they can be treated as an additive noise on the extension function $h(x)$:

$$h_n(x) = h(x) + I(x)n_{\mathrm{m}}(x, \tau), \tag{10}$$

with a yet unknown intensity function, and where dependency on variable τ indicates the different realizations on different source elements.

Substitution of the noisy driving function and extension function into equation (8) reveals that the radiated field can be written as the sum of four components:

- the ideal noiseless sound field of the virtual source: $Q(x) * h(x) * G(x, y, 0)$
- radiated noise resulting from stochastic sensitivity: $Q(x)n_s(x) * (h(x) * G(x, y, 0))$
- radiated noise resulting from the mechanical noise: $(Q(x) * I(x)n_{\mathrm{m}}(x, \tau)) * G(x, y, 0)$
- a combined effect of the two noise components: $Q(x)n_s(x) * I(x)n_{\mathrm{m}}(x, \tau) * G(x, y, 0)$

In the followings it is investigated, how the intensity of these radiated noise components is distributed in the listener's plane.

4 EFFECTS OF STOCHASTIC SENSITIVITY

In the current section we investigate the properties of the radiated noise, originating from stochastic sensitivity for the case of a synthesized virtual monopole with special attention to the intensity distribution over the listener's plane.

As most physical random processes, the sensitivity noise is influenced by many random parameters, thus its distribution can be considered to be Gaussian: $n_{\mathrm{s}}(x) \in \mathcal{N}(0, \sigma_{\mathrm{s}}^2)$. It is obvious, that the sensitivity noise is uncorrelated, since there's no dependency between the sensitivity of different source elements. The autocorrelation of this component is therefore $R_{n_{\mathrm{s}}n_{\mathrm{s}}}(\tau) = \sigma_{\mathrm{s}}^2\delta(\tau)$. It can be easily proven that the resulting multiplicative noise component $n'_{\mathrm{s}}(x) = Q(x)n_{\mathrm{s}}(x)$ (from equation (9)) is a *non-stationary* white noise with the autocorrelation function $R_{n'_{\mathrm{s}}n'_{\mathrm{s}}}(x, \tau) = \sigma_{\mathrm{s}}^2|Q(x)|^2\delta(\tau)$.

From equation (8) the radiated noise, originating from the stochastic sensitivity can be written as

$$N_s(x, y) = n'_{\mathrm{s}}(x) * (h(x) * G(x, y)) = n'_{\mathrm{s}}(x) * \int_{-\infty}^{\infty} h(x - x_0)G(x_0, y)\mathrm{d}x_0. \tag{11}$$

Here the filtered Green's function describes one extended element of the secondary source distribution. In the frequency range of interest ($f < 1.5$ kHz) it is feasible to assume that the support of the source extension function is smaller than the wavelength of the Green's function, so that over its support $G(x, y)$ is approximately constant. Clearly, this approximation is valid at low frequencies, in the far-field of the secondary source distribution, especially in front of the virtual sound source (at $x = x_s$), where the Green's function is stationary. Using the assumption the convolution can be transformed into a multiplication and by denoting the integral of the spatial extension function by $E_s = \int_{-\infty}^{\infty} h(x)\mathrm{d}x$ the filtered Green's function reads

$$h(x) * G(x, y) \approx E_s G(x, y). \tag{12}$$

Numerical simulations, carried out by utilizing the FEM loudspeaker model also confirmed the validity of this approximation. As a result the radiated noise is written as

$$N_s(x, y) = E_s \int_{-\infty}^{\infty} Q(\tau)n_{\mathrm{s}}(\tau)G(x - \tau, y)\mathrm{d}\tau. \tag{13}$$

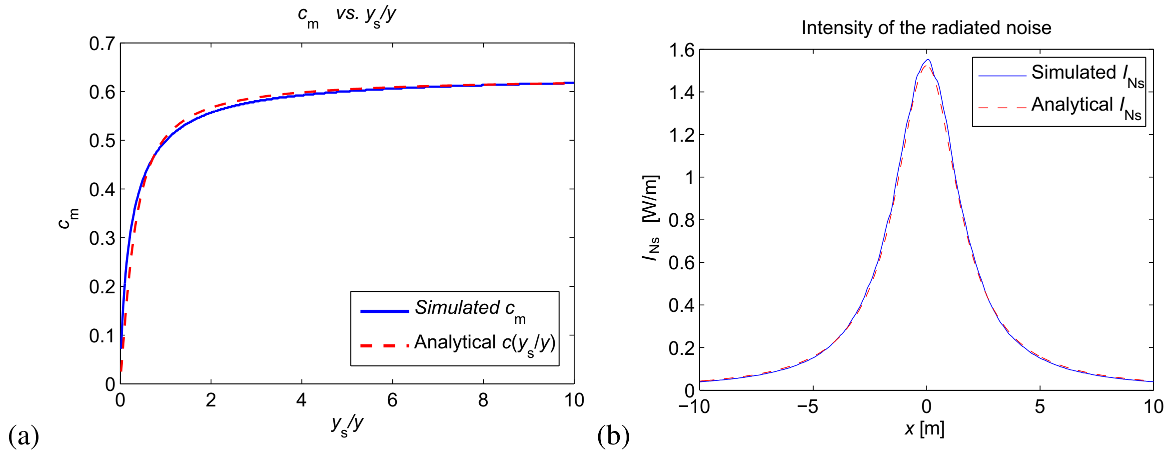


Figure 3: (a) Comparison of simulated and analytical c_{m} approaching factor and (b) intensity of the radiated noise, with simulation parameters: $y_s = 0.5$ [m], $y_{\mathrm{ref}} = 1$ [m], $y = 1.5$ [m], $\sigma_s = 1$, ($c_{\mathrm{m}} = 0.324$)

We are interested in the intensity distribution of the radiated noise, which is by definition $I_N(x, y) = \mathrm{E}\left(|N(x, y)|^2\right)$. It can be easily proven that the intensity distribution is

$$I_{N_s}(x, y) = \sigma_s^2 E_s^2 |Q(x)|^2 * |G(x, y)|^2. \tag{14}$$

The convolution can be carried out analytically, the derivation can be found in the appendix. As a final result we obtain

$$I_{N_s}(x, y) = \sigma_s^2 E_s^2 \frac{c_{\mathrm{m}} y_s + y}{y_s + y_{\mathrm{ref}}} \frac{y_{\mathrm{ref}}}{y} \frac{k/\pi}{r'^2}, \tag{15}$$

where $r' = \sqrt{x^2 + (y + c_{\mathrm{m}} y_s)^2}$ and

$$c_{\mathrm{m}}(y_s, y) \approx 0.63 \frac{2}{\pi} \mathrm{atan}\left(\pi \frac{y_s}{y}\right). \tag{16}$$

It is important to note that for a fixed listener position y the intensity function (15) describes the intensity of the sound field of a point source. This phantom source is located closer to the secondary source than the original virtual point source. The actual approaching factor of the phantom source to the secondary source array, denoted by c_{m} varies as the function of the listener position and the virtual source position from a factor of 0 to about 0.63: far from the secondary source distribution, the position of the phantom source gets close to the secondary source line.

In figure 3 (b) the result of Monte Carlo simulations is shown, comparing the average intensity of the radiated noise, evaluating equation (11) directly with different noise realizations and the result of analytical formulation, given by (15). The simulation were carried out by averaging 5000 realizations. As it can be seen the analytical approximation of the average radiated intensity shows a very good match with the results of the Monte Carlo simulations.

5 EFFECTS OF THE MECHANICAL NOISE

In the followings the direct effects of the stochastic mechanical properties of secondary sources are investigated.

As it was already pointed out, the noise originating from mechanical anisotropy may be regarded as an additive noise to the loudspeaker extension function. To gain insight into the properties of the mechanical noise stochastic mechanical properties were added to the mechanical FEM loudspeaker model.

As the part of investigation slightly varying material density was assumed by adding exponentially correlated Gaussian noise to the mass matrix. Correlated, periodic noise was also added to the stiffness matrix for the suspension elements, symbolizing slightly varying suspension stiffness. Figure 4 (a) shows one realization of the additive noise to the loudspeaker material density.

Monte Carlo simulations were carried out, evaluating the resulting loudspeaker surface velocity and calculating the one-dimensional loudspeaker extension function by direct evaluation of equation (7). By performing 500 simulations for different noise realization it was found, that the mechanical noise – denoted by $n_m(x,\tau)$ – is non-stationary, with the intensity function proportional to the extension function, thus

$$h'(x) = h(x) + h(x)n_m(x,\tau), \tag{17}$$

and its autocorrelation function is described by $R_{n_m n_m}(x,\tau) = R_{n_m n_m}(x)\delta(\tau)$, meaning that the noise is uncorrelated among the different loudspeakers. The average intensity of the mechanical noise can be seen in figure 4 (b).

Properties of the radiated noise:

The radiated noise can be calculated from the convolution

$$N_m(x,y) = (h(x)n_{\mathrm{m}}(x,\tau) * Q(x)) * G(x,y,0) = G(x,y) * \int_{-\infty}^{\infty} Q(\tau)h(x-\tau)n(x-\tau,\tau)\mathrm{d}\tau. \tag{18}$$

In the integral the kernel of convolution – the non-stationary noise – changes continuously with the convolution spatial shift, which indicates that there's a different noise realization on each element of the secondary source distribution. This means that the radiated field is obtained as the result of a non-stationary convolution. For non-stationary convolution the associative property of stationary convolution does not hold, meaning that in order to obtain a correct result convolutions have to be carried out sequentially.

In the equation $n'_m(x) = \int_{-\infty}^{\infty} Q(\tau)h(x-\tau)n(x-\tau,\tau)\mathrm{d}\tau$ results in one noise realization, as an additive noise on the driving function $Q(x)$. The intensity of this noise can be expressed analytically (by exploiting that the noise is uncorrelated in direction τ):

$$I_{n'_m}(x) = \sigma_m^2 |Q(x)^2| * |h(x)|^2, \tag{19}$$

where $\sigma_m^2 = R_{mm}(0)$ is the standard deviation of the original mechanical noise, infused into the FEM model. Here function $|Q(x)|^2$ is a smooth function of x, thus it is feasible to suggest that it is approximately constant over the interval, specified by the support of $|h(x)|^2$, therefore the same approximation can be applied as it was given in the previous section:

$$I_{n'_m}(x) = \sigma_m^2 E_m^2 |Q(x)^2|, \tag{20}$$

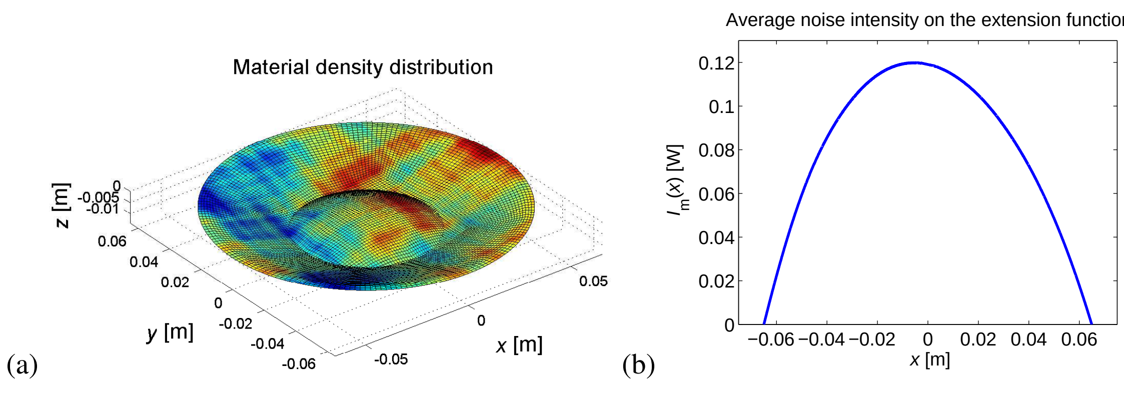


Figure 4: (a) Additive noise loudspeaker material density (b) Intensity of resultant additive noise on the extension function $h(x)$ at 100 Hz

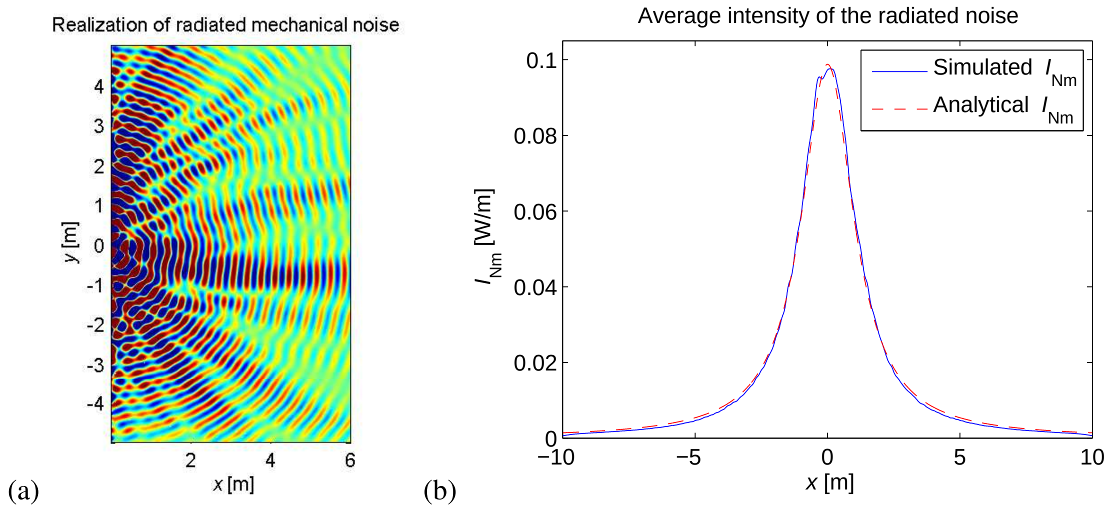


Figure 5: (a) One realization of radiated noise, originating from the mechanical noise and (b) intensity of the radiated noise, with simulation parameters: $y_s = 1$ [m], $y_{\text{ref}} = 1$ [m], $y = 1$ [m], $\sigma_s = 1$ ($c_{\text{m}} = 0.4$)

where $E_m^2 = \int_{-\infty}^{\infty} |h(x)|^2 \mathrm{d}x$. Note that whilst in the previous section E_s^2 was the square of the average value of the loudspeaker extension function, here E_m^2 is the total energy of the function, thus $E_m^2 \geq E_s^2$ holds.

It can be proven, that the autocorrelation function of $n'_m(x)$ is specified by $h(x)$, so that the supports of the two functions are equal. As the autocorrelation function is of small support, it is feasible to approximate the noise as a non-stationary white noise: $R_{n'_m n'_m}(x, \tau) \approx \sigma_m^2 E_m^2 |Q(x)^2| \delta(\tau)$.

Using this, the radiated noise can be written as:

$$N_m(x, y) = G(x, y) * n'_m(x). \tag{21}$$

For the equation the same procedure can be carried out as it was shown in the previous section for sensitivity noise and finally the intensity of radiated noise, similarly to that of the sensitivity noise reads:

$$I_{N_m}(x, y) = \sigma_m^2 E_m^2 |Q(x)^2| * |G(x, y)|^2. \tag{22}$$

The analytic expression for the radiated intensity can be given in the same manner as for the sensitivity noise, given in the appendix, thus for the average intensity distribution the following final result was obtained:

$$I_{N_m}(x, y) = \sigma_m^2 E_m^2 \frac{c_{\text{m}} y_s + y}{y_s + y_{\text{ref}}} \frac{y_{\text{ref}}}{y} \frac{k/\pi}{r'^2}, \tag{23}$$

where $r' = \sqrt{x^2 + (y + c_{\text{m}} y_s)^2}$ and c_{m} is given by equation (16).

The result indicates that not only the noise due to the stochastic sensitivity, but also the mechanical noise seems to originate from a phantom source, located closer to the secondary source distribution. In figure 5 (a) one realization of the radiated mechanical noise can be examined, while figure 5 (b) depicts the comparison of the result of Monte Carlo simulation and the analytical formula (23) for the average radiated noise intensity. As it can be seen, again a very good match can be observed, meaning that the assumptions made for the approximations are correct over a wide parameter range.

6 CONCLUSION

In the present contribution a treatise on stochastic sound field reproduction was given. In the presented model spatially extended source elements were considered as secondary source elements, modeling real-life loudspeakers. Extending the traditional model we also took the stochastic properties of real-life sound sources into consideration. Here stochastic loudspeaker sensitivity and stochastic mechanical properties were examined in details.

For the intensity of the radiated noise originating from the stochastic sensitivity and stochastic material properties an analytic formula can be given. For the sake of finding a closed form for the average intensity of the radiated noise a well-usable approximation was given for the modified Bessel-function, applying exponential function with a varying exponent.

The main finding of the research was that the radiated noise components seem to originate from a point source with a stochastic directivity, closer to the secondary source distribution than the original virtual source. The approaching factor is not constant, it is the function of the original virtual source position and the listener's position. For this approaching function an analytic approximation was given, obtained by numerical error minimization.

In the present proceeding only a part of the research, treating stochastic sound field synthesis was presented. Giving a mathematical formulation for the combined effect of sensitivity noise and mechanical noise is the topic of a future work. Besides this, both the sensitivity noise and mechanical noise can be considered to be frequency dependent. To investigate the effects of the sensitivity noise in case of synthesizing wave fronts emitted by virtual sources time domain analysis is needed. This aspect of examination is also yet to be carried out.

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APPENDIX

Evaluation of convolution for the radiated noise:

In equation (14) energy of the three-dimensional Green's function and the 2.5-dimensional driving functions are:

$$|G(x,y)|^2 = \frac{1}{16\pi^2}\frac{1}{x^2+y^2}, \qquad |Q(x,y)|^2 = \frac{k}{2\pi}\frac{y_{\mathrm{ref}}y_s^2}{y_s+y_{\mathrm{ref}}}\frac{1}{(x^2+y_s^2)^{\frac{3}{2}}}. \tag{24}$$

The convolution was carried out in the spectral domain, as due to the properties of Fourier-transform:

$$I_{N_s}(k_x,y) = \sigma_s^2 \mathrm{E}_h^2 \mathcal{F}_x\left(|Q(x,y)|^2\right)\mathcal{F}_x\left(|G(x,y)|^2\right), \tag{25}$$

where x and k_x are Fourier-transform pairs.

Here, without derivation the required Fourier-transforms are

$$\mathcal{F}_x\left(|G(x,y)|^2\right) = \sqrt{\frac{\pi}{2}}\frac{\mathrm{e}^{-y|k_x|}}{y}, \quad \mathcal{F}_x\left(|Q(x)|^2\right) = \frac{k}{\sqrt{2\pi^3}}\frac{y_{\mathrm{ref}}y_s}{y_s+y_{\mathrm{ref}}}|k_x|K_1(y_s|k_x|), \tag{26}$$

where $K_1()$ is the modified Bessel function of the second kind.

Approximation of the modified Bessel function:

In order to carry out the inverse Fourier-transform of equation (25) analytically, the Bessel function is approximated by an exponential function:

$$y_s|k_x|K_1(y_s|k_x|) \approx \mathrm{e}^{-cy_s|k_x|} \tag{27}$$

The error term was defined so that the error of the spectral product was taken into consideration:

$$\epsilon(k_x,c) = \left|\int_{-\infty}^{\infty}|G(k_x,y)|^2\left(|k_x|K_1(y_s|k_x|) - \frac{\mathrm{e}^{-cy_s|k_x|}}{y_s}\right)\mathrm{d}k_x\right|. \tag{28}$$

Note that this error definition will ensure minimal error in front of the virtual sound source, at $x=0$, as the integral part is the definition of the inverse Fourier transform of the error term at $x=0$. The error term was minimized numerically by calculating parameter c, for which: $\min\left(\epsilon(k_x,c)\right) = \epsilon(k_x,c_{\mathrm{m}}) \approx 0$. It was found that the constant c_{m} at which the expression takes its minimum value is a function of ratio $\frac{y_s}{y}$.

The result of numerical estimation of c_{m} is shown in figure 3 (a) with continuous blue line. An analytical approximation of this function was found, shown in the figure by red dashed line:

$$c_{\mathrm{m}}(y_s,y) \approx 0.63\frac{2}{\pi}\mathrm{atan}\left(\pi\frac{y_s}{y}\right). \tag{29}$$

Using the approximation the spectrum of the squared energy of the driving function is written as:

$$\mathcal{F}_x\left(|Q(x)|^2\right) \approx \frac{k}{\sqrt{2\pi^3}}\frac{y_{\mathrm{ref}}}{y_s+y_{\mathrm{ref}}}\mathrm{e}^{-c_{\mathrm{m}}y_s|k_x|}, \tag{30}$$

where c_{m} is given by equation (29).

Evaluation of the radiated noise intensity:

Using the aforementioned, the spectrum of the intensity of the radiated noise can be given by

$$I_{N_s}(k_x,y) = \sigma_s^2 E_s^2 \frac{k}{2\pi}\frac{y_{\mathrm{ref}}}{y_s+y_{\mathrm{ref}}}\frac{\mathrm{e}^{-|k_x|(y+c_{\mathrm{m}}y_s)}}{y}. \tag{31}$$

The inverse Fourier-transform can be carried out by utilizing the definite integral of exponential functions. By introducing $r' = \sqrt{x^2+(y+c_{\mathrm{m}}y_s)^2}$, as a final result for the average intensity distribution we obtain

$$I_{N_s}(x,y) = \sigma_s^2 E_s^2 \frac{c_{\mathrm{m}}y_s+y}{y_s+y_{\mathrm{ref}}}\frac{y_{\mathrm{ref}}}{y}\frac{k/\pi}{r'^2}. \tag{32}$$

Evaluation of filtered mechanical noise correlation:

As we could see in the related section the mechanical noise acts as a non-stationary additive noise on the driving function, described by the non-stationary convolution:

$$n'_m(x) = \int_{-\infty}^{\infty} Q(\tau)h(x-\tau)n_m(x-\tau,\tau)\mathrm{d}\tau. \tag{33}$$

By definition, the autocorrelation function is written as $R_{n'_m n'_m}(x,\xi) = \mathrm{E}\left(n'_m(x)n'_m(x-\xi)\right)$:

$$R_{n'_m n'_m}(x,\xi) = \mathrm{E}\Big(\int_{-\infty}^{\infty} Q(\tau_1)h(x-\tau_1)n_m(x-\tau_1,\tau_1)\mathrm{d}\tau_1 \int_{-\infty}^{\infty} \overline{Q}(\tau_2)\overline{h}(x-\tau_2-\xi)\overline{n}_m(x-\tau_2-\xi,\tau_2)\mathrm{d}\tau_2\Big). \tag{34}$$

By changing the sequence of integration and expectation, and by applying that the expected value of a deterministic function is the function itself:

$$R_{n'_m n'_m}(x,\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\tau_1)\overline{Q}(\tau_2)h(x-\tau_1)\overline{h}(x-\tau_2-\xi)\mathrm{E}\left(n_m(x-\tau_1,\tau_1)\overline{n}_m(x-\tau_2-\xi,\tau_2)\right)\mathrm{d}\tau_1\mathrm{d}\tau_2. \tag{35}$$

by definition: $\mathrm{E}\left(n_m(x-\tau_1,\tau_1)\overline{n}_m(x-\tau_2-\xi,\tau_2)\right) = R_{n_m n_m}(\tau_2-\tau_1+\xi,\tau_1-\tau_2)$:

$$R_{n'_m n'_m}(x,\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\tau_1)\overline{Q}(\tau_2)h(x-\tau_1)\overline{h}(x-\tau_2-\xi)R_{n_m n_m}(\tau_2-\tau_1+\xi,\tau_1-\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2. \tag{36}$$

We know that the autocorrelation of the original mechanical noise is a Dirac function in its second dimension:

$$R_{n'_m n'_m}(x,\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\tau_1)\overline{Q}(\tau_2)h(x-\tau_1)\overline{h}(x-\tau_2-\xi)R_{n_m n_m}(\tau_2-\tau_1+\xi)\delta(\tau_1-\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2. \tag{37}$$

We may exploit the sifting property of the Dirac function, thus integration according $\mathrm{d}\tau_1$ will sift out the value of integrand at $\tau_1 = \tau_2 = \tau$:

$$R_{n'_m n'_m}(x,\xi) = \int_{-\infty}^{\infty} Q(\tau)\overline{Q}(\tau)h(x-\tau)\overline{h}(x-\tau-\xi)R_{n_m n_m}(\xi)\mathrm{d}\tau. \tag{38}$$

$$R_{n'_m n'_m}(x,\xi) = R_{n_m n_m}(\xi)\int_{-\infty}^{\infty} |Q(\tau)|^2 h(x-\tau)\overline{h}(x-\tau-\xi)\mathrm{d}\tau. \tag{39}$$

We may use the denotation $h'(x,\xi) = h(x)\overline{h}(x-\xi)$, so the autocorrelation will read:

$$R_{n'_m n'_m}(x,\xi) = R_{n_m n_m}(\xi)|Q(x)|^2 * h'(x,\xi). \tag{40}$$

We know, that $h(x) = 0$, if $x \notin [-\frac{L}{2};\frac{L}{2}]$, thus the support of the extension function is L, the diameter of the loudspeaker. It means that function $h'(x,\xi) = h(x)\overline{h}(x-\xi) \equiv 0$ if $\xi \notin [-L;L]$, while in region $\xi \in [L;L]$ the maximum (and the energy) of the function $h'(x,\xi)$ decays rapidly with increasing space lag ξ.

As this will also mean that $R_{n'_m n'_m}(x,\xi) \equiv 0$, if $|\xi| > L$ the correlation lenght of the noise realization is small, thus it is feasible to substitute the weakly correlated noise with a non-stationary white noise, so that

$$R_{n'_m n'_m}(x,\xi) = R_{n_m n_m}(0)|Q(x)|^2 * |h(x)|^2\delta(\xi). \tag{41}$$

By utilizing that $|h(x)|^2$ is of small support the convolution can be transformed into a multiplication: by denoting $|h(x)|^2 = E_m^2$ and $\sigma_m^2 = R_{n_m n_m}(0)$ the equation reads:

$$R_{n'_m n'_m}(x,\xi) = \sigma_m^2 E_m^2 |Q(x)|^2\delta(\xi). \tag{42}$$