Prefiltering the wave field synthesis operators - anti-aliasing and source directivity

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Outline

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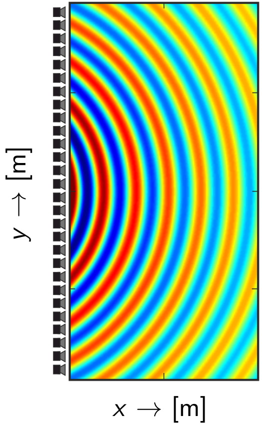
  - Spectral division method (SDM)
- Research activity
  - Comparison fo WFS and SDM driving functions
  - Effects of linear filtering of the synthesis operators
- Applications of linear filtering:
  - new method for synthesizing directive sources
  - proper filter design to avoid spatial aliasing
- Conclusion

Motivation

Synthesis of a virtual monopole



Problem with stereophonic techniques:

- Perfect surround sound only at the sweet spot

Aim of sound field reconstruction:

- Physically recreating virtual wave fields, wave fronts with properly driven densely spaced loudspeakers

Applications:

- Entertainment (Cinemas, sound enhancement in theatres)
- Active noise control

Motivation

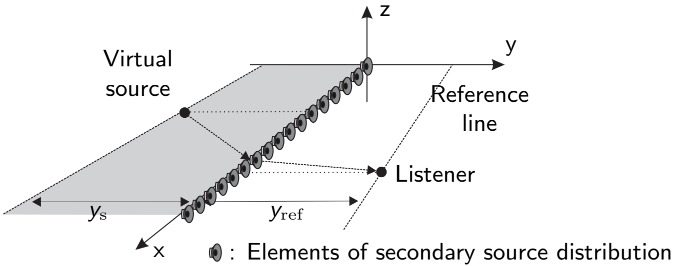
Phenomena, described as filtering problems physically arise from the theory of sound field reconstruction

Effects of linear filtering of the loudspeaker driving functions have not been investigated

Objectives:

- Examine the effects of filtering on the reconstructed sound field
- Give a utilizeable physical interpretation for linear filtering
- Utilize the presented technique for improved synthesis of directional sources
- Find optimal filter design for anti-aliasing filtering

Theory of sound field reconstruction

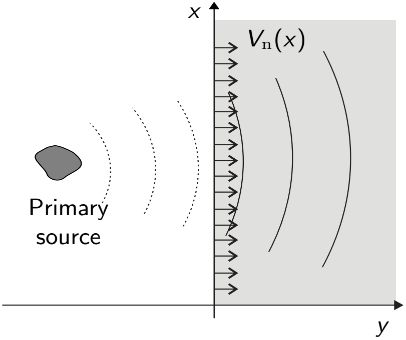


General sound field reconstruction problem:

$$P(\mathbf{x},\omega)_{\mathrm{synth}} = \int_{-\infty}^{\infty} Q(\mathbf{x}_0,\omega)G(\mathbf{x}|\mathbf{x}_0,\omega)\mathrm{d}x \stackrel{!}{=} P(\mathbf{x},\omega)_{\mathrm{virtual}}$$

Wave field synthesis

- Derivation of driving functions in spatial domain
- Rayleigh integrals:
  Sound field of a sound source via boundary conditions, in case of linear boundary
  $\downarrow$
  Rayleigh I. contains explicitly the driving functions
- Approximations $\rightarrow$ perfect synthesis only on reference line



$$P(\mathbf{x},\omega)_{\mathrm{primary}} = \int_{-\infty}^{\infty} \underbrace{-\mathrm{j}\omega\rho_0 V_{\mathrm{n}}(\mathbf{x}_0,\omega)}_{Q(\mathbf{x}_0,\omega)} G(\mathbf{x}|\mathbf{x}_0,\omega)\mathrm{d}x$$

Spectral Division Method

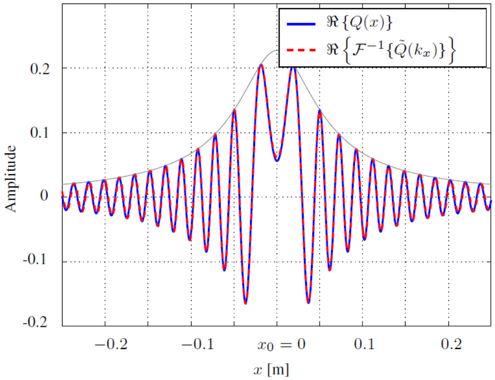
- Derivation of driving functions in spectral domain
- Fourier-transform along x-axis: $x \xrightarrow{\mathcal{F}} k_x = k \sin\varphi$
- The general reconstruction integral, written along the reference line represents a convolution along the synthesis line:

$$P(x, y_{\mathrm{ref}}) = \int_{-\infty}^{\infty} Q(x_0) G_{\mathrm{3D}}(x - x_0, y_{\mathrm{ref}}) \mathrm{d}x = Q(x) * G_{\mathrm{3D}}(x, y_{\mathrm{ref}})$$

- If spectra of the virtual sound field and secondary monopole known on reference line:

$$\tilde{Q}(k_x) = \frac{\mathcal{F}\left(P(x, y_{\mathrm{ref}})\right)}{\mathcal{F}\left(G(x, y_{\mathrm{ref}})\right)} = \frac{\tilde{P}(k_x, y_{\mathrm{ref}})}{\tilde{G}(k_x, y_{\mathrm{ref}})}$$

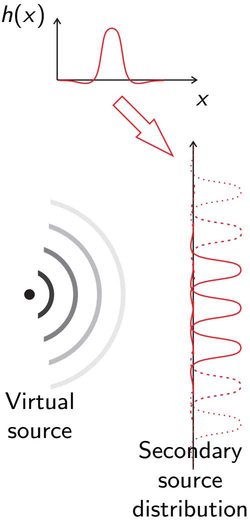
Comparison of WFS and SDM driving functions



Result: in the far-field of monopole ($kr \gg 1$) WFS $\approx$ SDM

$$\mathcal{F}^{-1}\left(Q_{\mathrm{SDM}}(k_x, \omega)\right) \approx Q_{\mathrm{WFS}}(x, \omega) = Q(x)$$

Linear filtering of the driving functions

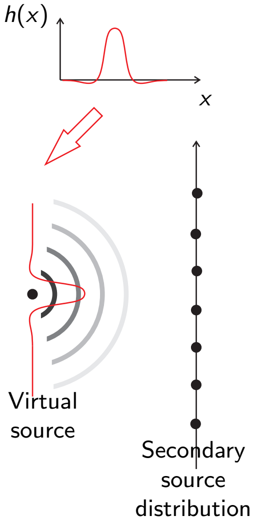


Linear filter is defined with it impulse response ($h(x)$) or transfer function ($\tilde{H}(k_x)$)

Conclusions:

- Filtering $Q(x) \equiv$ synthesis with extended secondary sources

Linear filtering of the driving functions



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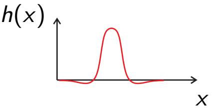
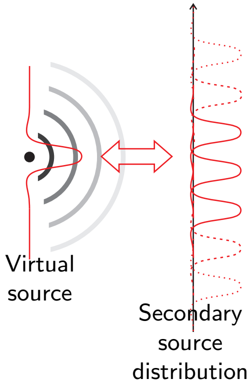
- Filtering $Q(x) \equiv$ synthesis with extended secondary sources
- Filtering $Q(x) \equiv$ synthesis of the field of an extended virtual source

Linear filtering of the driving functions





Linear filter is defined with it impulse response ($h(x)$) or transfer function ($\tilde{H}(k_x)$)

Conclusions:

- Filtering $Q(x) \equiv$ synthesis with extended secondary sources
- Filtering $Q(x) \equiv$ synthesis of the field of an extended virtual source
- Virtual source extension and secondary source extension are interchangeable
- Extension of virtual, or secondary source elements is defined by the filter impulse response

Application: Synthesizing directive sources

- Spatial extension (normal velocity) of the virtual source is given $\rightarrow$ impulse response of the filter is defined explicitly

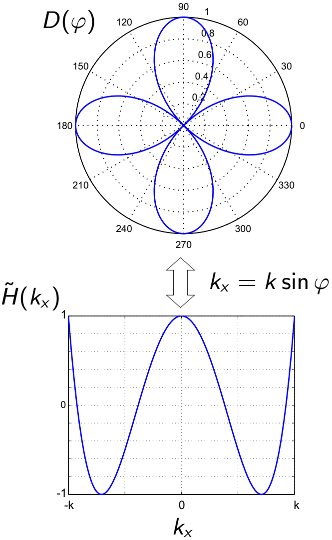
$$V_{\mathrm{n}}(x_{\mathrm{S}}) \propto h(x)$$

- In the far-field the extended source seems as a directive point source:

$$P(\mathbf{x}) = D(\varphi) G(\mathbf{x}|\mathbf{x}_s) \propto \tilde{V}_{\mathrm{n}}(k_x) G(\mathbf{x}|\mathbf{x}_s)$$

- Directivity function of the virtual source is given $\rightarrow$ transfer function of the filter is defined explicitly:

$$D(\varphi) \xrightarrow{k_x = k \sin\varphi} \tilde{H}(k_x)$$



Example: synthesizing a quadrupole

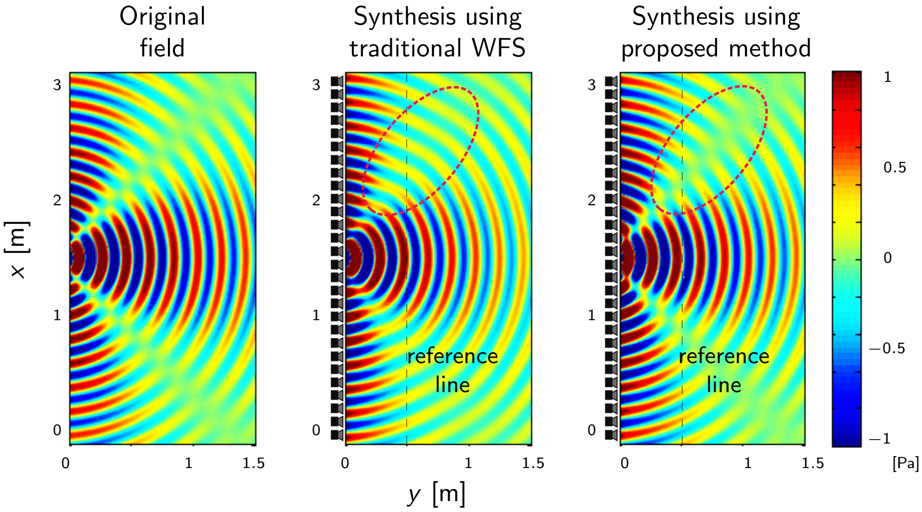
- Directivity function is given: $D(\varphi) = \cos(2\varphi)$
- Transfer function of the linear filter: $\tilde{H}(k_x) = \cos\left(2\arcsin\frac{k_x}{k}\right)$
- Impulse response of the filter: $h(x) = \delta(x) - \frac{2\delta''(x)}{k^2}$
- The proposed (lineary filtered) driving functions:

$$Q_{\text{quad}}(x) = h(x) * Q_{\text{mono}}(x) = Q_{\text{mono}}(x) - \frac{2}{k^2} Q''_{\text{mono}}(x)$$

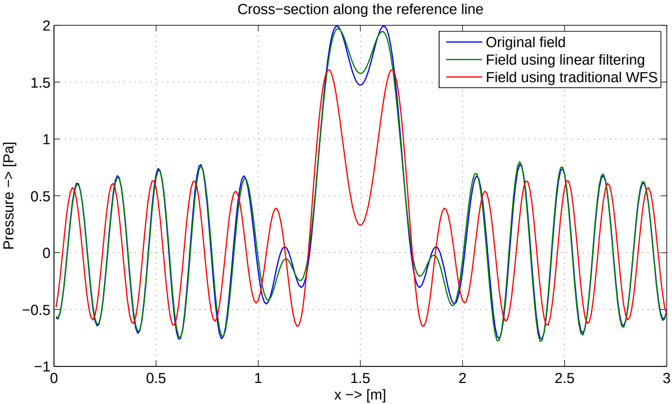
- Traditional WFS driving functions for a virtual quadrupole:

$$Q_{\text{quad}}^{\text{trad}}(x) = D(\varphi) Q_{\text{mono}}(x)$$

Example: synthesizing a quadrupole

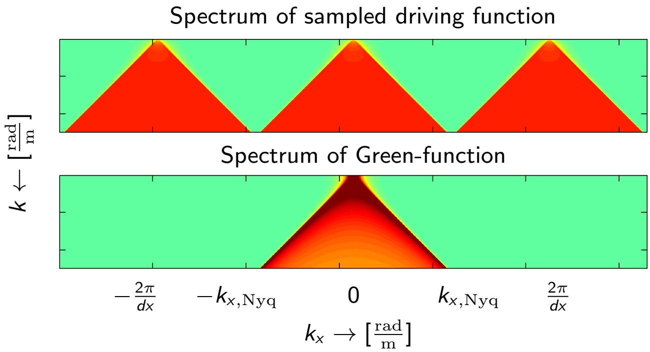


Example: synthesizing a quadrupole



- Traditional synthesis: great amplitude and phase errors
- Proposed method: only slight amplitude errors

Effects of spatial sampling



- Continuous source distribution $\rightarrow$ discrete loudspeakers $\equiv$ spatial sampling of the secondary distribution
- Mathematical model: applying sampled driving function

Avoiding spatial aliasing

- Effect of aliasing: undesired plane wave components

$$\downarrow$$

Spatial-bandlimiting is needed

- Reproduction filter:
  - Spatial low pass filtering by the secondary source distribution
  - Directive source elements act as spatial low pass filter
- Anti-aliasing filter:
  - Pre-filtering of $Q(x)$ is needed
  - Proper low-pass filter design is possible

Avoiding spatial aliasing

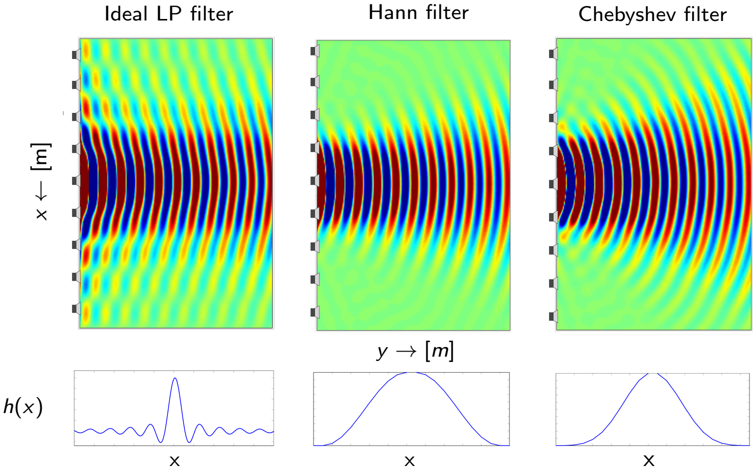
- Assuming ideal stop band attenuation:

> The virtual source extension is defined by the resultant of anti-aliasing and reconstruction filters

- Filter design considerations for a virtual monopole:
  - The resultant impulse response should be a Dirac-delta
  - Narrow impulse response $\rightarrow$ low stop-band attenuation
  - Sharp transition on cut-off k_x $\rightarrow$ wide virtual source
  - Compromise is needed

Examples for anti-aliasing filtering

Aim: synthesize a virtual monopole on extended area, applying discrete sources



Summary and conclusion

Main findings

- WFS and SDM techniques are proven to be equivalent for a virtual monopole in the far-field
- linear filtering of the driving functions can be interpreted as synthesis, applying spatially extended / directive secondary or virtual sources

Applications of linear filtering:

- Improved method for synthesis of directive sources
- Proper anti-aliasing filtering for discrete secondary source distribution

Further possibilities

- Analytical driving function for arbitrary directive sound sources based on multipole expansion
- Examine the level of audible aliasing effects to optimize the anti-aliasing filter

Thank you for your attention!