A Generalized Wave Field Synthesis Framework
with Application for Moving Virtual Sources
PhD defense

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2019. November 22.

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Laboratory of
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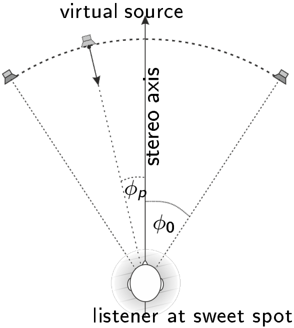
What is sound field synthesis (SFS)?



- Aim of sound field reproduction: to create the impression of a desired audio scene
- Stereophony:
  - reconstructs binaural cues
    - Interaural time difference
    - Interaural level difference
  - correct sound localization only in the *sweet spot*

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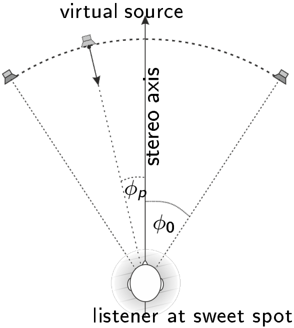

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  - reconstructs binaural cues
    - Interaural time difference
    - Interaural level difference
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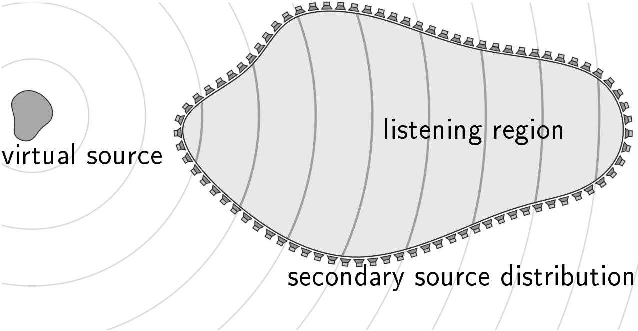
- Goal of sound field synthesis: to reconstruct physical properties of desired sound field **over an extended region**

↓

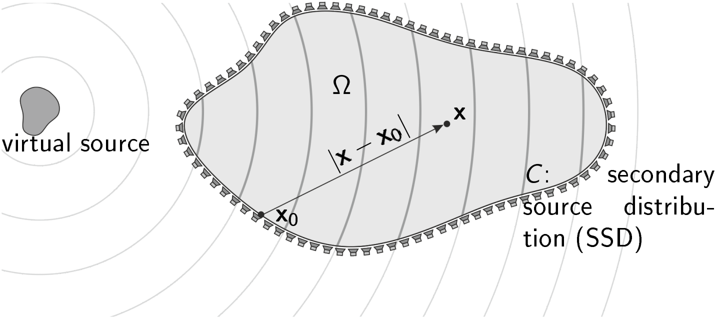
- Perfect localization inherently ensured

What is sound field synthesis (SFS)?



- Goal: physical reproduction of a target/**virtual** sound field over an extended region (horizontal plane)
- Densely spaced loudspeaker contour: **secondary source distribution (SSD)**
- Problem: to find the optimal loudspeaker **driving functions**

What is sound field synthesis (SFS)?



- Synthesized field: convolutional integral

$$P(\mathbf{x},\omega) = \oint_C D(\mathbf{x}_0,\omega)\, G(\mathbf{x}-\mathbf{x}_0,\omega) \mathrm{d}s(\mathbf{x}_0),$$

  - $P(\mathbf{x},\omega)$: prescribed target/virtual sound field
  - $G(\mathbf{x}|\mathbf{x}_0,\omega)$: field of the secondary source elements
  - $D(\mathbf{x}_0,\omega)$: driving function to be found

Solutions for the SFS inverse problem

- Explicit solution
  - direct solution of the inverse problem in the spectral domain
  - compact formula rarely available
  - exists only for particular geometries:
    - linear SSD: **Spectral Division Method (SDM)**
    - circular/spherical SSD: **Nearfield Compensated Higher Order Ambisonics (NFC-HOA)**
- Implicit solution: **Wave Field Synthesis (WFS)**
  - based on the Huygens' principle
  - relies on boundary integral representation of sound fields, containing the required driving function *implicitly*
  - central topic of the present dissertation

Outline

- Introduction
- Thesis group 1: Generalization of Wave Field Synthesis theory
- Thesis group 2: Spatial explicit driving functions and WFS equivalence
- Thesis group 3: Wave Field Synthesis of moving point sources
- Thesis group 4: Synthesis of moving sources in the wavenumber domain
- Conclusion

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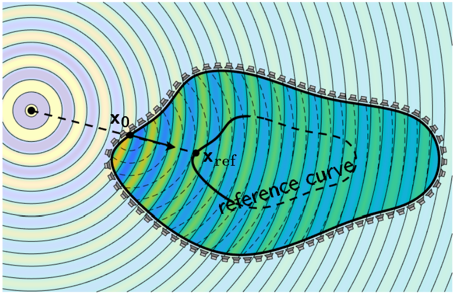
Thesis group I: Generalized WFS theory

- Goal: extraction of the 2D (contour) driving functions from the 3D (surface) Kirchhoff integral
- Method: asymptotic approximation by the stationary phase approximation (**high frequency conditions**)
- Introduced concept: the local wavenumber vector



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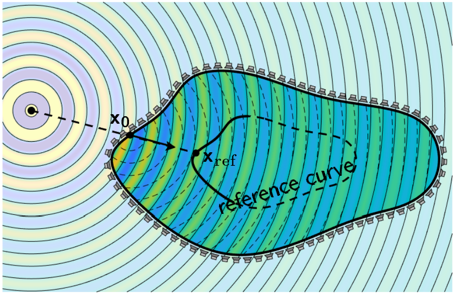


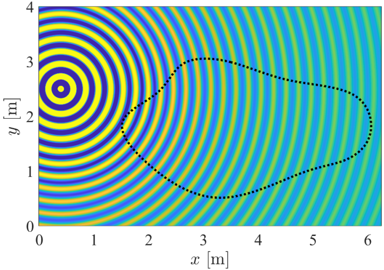
Result: generalized WFS loudspeaker driving functions for

- arbitrary virtual fields (propagating along the plane of synthesis)
- arbitrary SSD shapes
- arbitrary reference curve: contour of amplitude and phase correct synthesis, **defined via the local wavenumber vector**

Thesis group I: Generalized WFS theory

- Application example: synthesis of a virtual harmonic point source

Target field



Thesis group I: Generalized WFS theory

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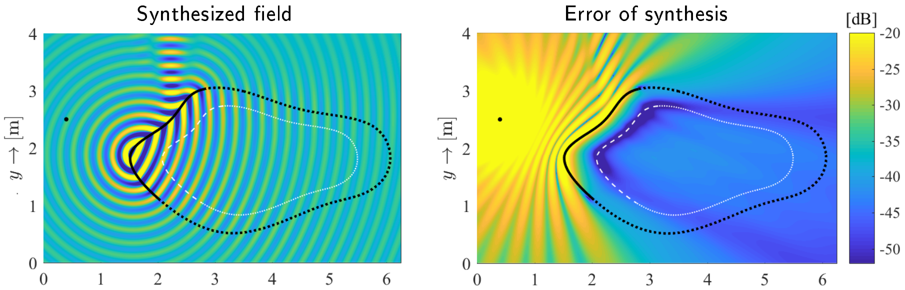

- Result:
  - phase correct inside the listening region (→ wavefront shape preserved)
  - amplitude error minimized over the reference curve

Thesis group I: Generalized WFS theory

Relation with previous approaches:

	Traditional WFS	Revisited WFS	Generalized WFS
arbitrary SSD shape	×	✓	✓
arbitrary virtual field	×	×	✓
arbitrary ref. curve	×	×	✓

- Summary: previous approaches are special cases of generalized WFS theory

- Example: generalized WFS for 3D virtual point source with linear SSD and linear reference curve = traditional WFS

Thesis group I: Unified WFS theory

Thesis group summary:

- Thesis I.1: Physical interpretation for the SPA of boundary integrals: wave front matching of the virtual field and the secondary sound fields [J2]
- Thesis I.2: 2.5D WFS driving functions for arbitrary virtual fields and SSD shapes [J2]
- Thesis I.3: Analytical expressions for the *reference curve* and the *referencing function* [J2]

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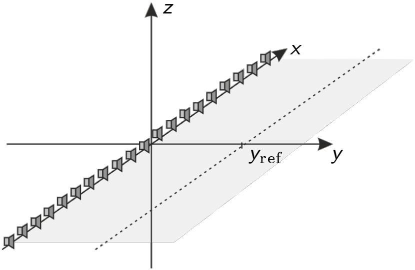
Thesis group II: Spatial explicit driving functions



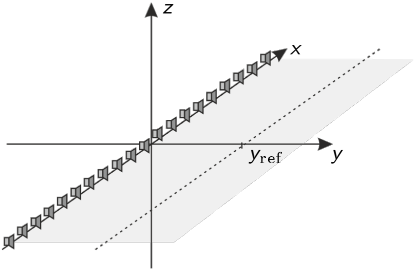
- Explicit driving functions: Fourier integral

$$D(x_0,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\tilde{P}(k_x,\omega)}{\tilde{G}(k_x,\omega)} \mathrm{e}^{-\mathrm{j}k_x x_0} \mathrm{d}k_x$$

- No general closed form → practical implementation infeasible

- Goal: spatial domain approximation of the Fourier integral
- Method: asymptotic evaluation by the stationary phase method

Thesis group II: Spatial explicit driving functions



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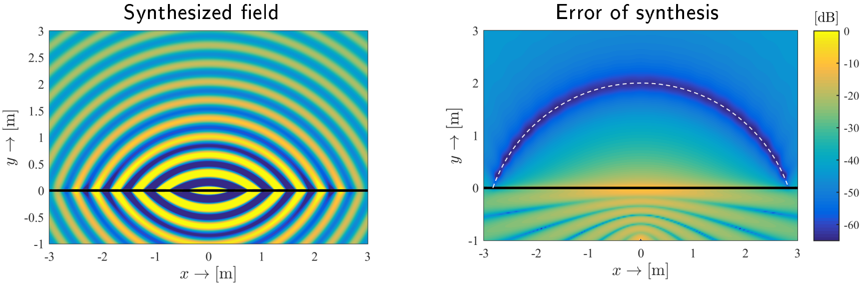
- Results:
    - Explicit driving functions in the spatial domain for
        - arbitrary virtual sources
        - arbitrary SSD shape
        - arbitrary reference curve, defined via the local wavenumber vector
    - Explicit and implicit solutions are equivalent for high frequencies
        - allows unified discussion of spatial aliasing phenomena

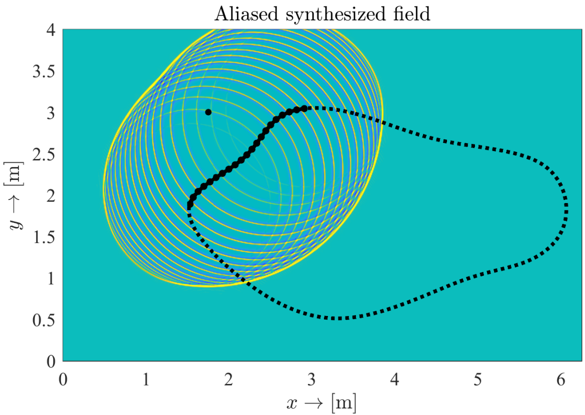
Thesis group II: Spatial explicit driving functions

- Example:
  - virtual field: 3D harmonic point source
  - SSD shape: linear
  - ref. curve: circle around the virtual source



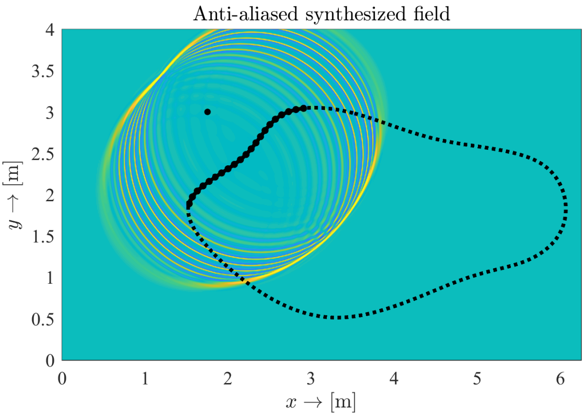
- Result
  - phase correct synthesis inside the listening region
  - minimal amplitude error over the reference circle

Thesis group II: Avoiding spatial aliasing



- Aliasing: high-pass filtered wavefronts following the intended virtual wavefront due to SSD discretization
- Local aliasing frequency: from asymptotic approximation of the explicit driving functions in terms of local wavenumber vector

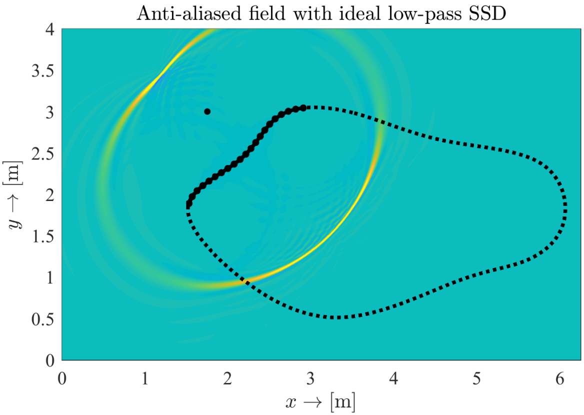
Thesis group II: Avoiding spatial aliasing



- Result: Anti-aliasing criterion: low-pass filtering below local aliasing frequency

$$D(\mathbf{x}_0, \omega) = 0, \qquad \omega \geq \frac{\pi}{\Delta x} \frac{c}{|\hat{k}_t^P(\mathbf{x}_0)|}$$

Thesis group II: Avoiding spatial aliasing



- Suppressing lateral aliasing waves: with directive loudspeakers

Thesis group II: Spatial explicit driving functions and WFS equivalence

Thesis group summary:

- Thesis II.1: Analytical SDM driving functions merely in the spatial domain, expressed in terms of the target sound field measured along an arbitrary reference curve [J4]
- Thesis II.2: Under high-frequency assumptions the explicit SDM and the implicit WFS driving functions are completely equivalent for an arbitrary target sound field [J4]
- Thesis II.3: Anti-aliasing criterion that can be implemented in practice by simple low-pass filtering of the loudspeaker driving signals [C10]

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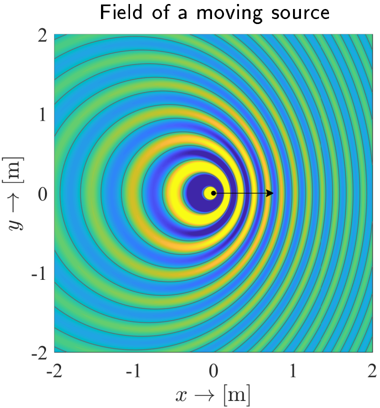
Thesis group III-IV: SFS of moving sources

- Complex application example: synthesis of moving sound sources
- Primary challenge: reconstruction of the Doppler effect



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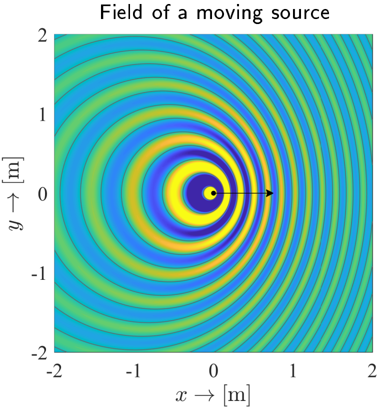
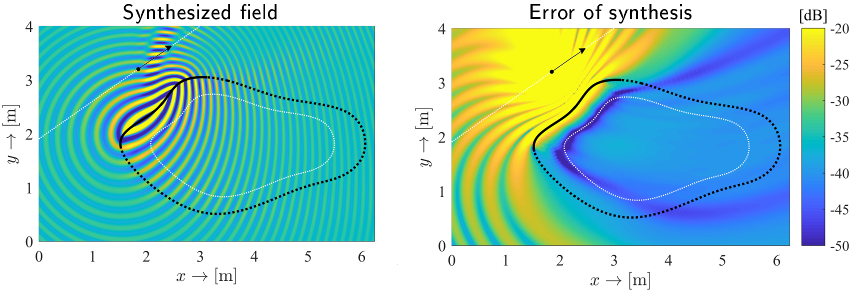


Result:

- Extension of the generalized SFS framework to include moving sources
- WFS driving function for sources, moving on an arbitrary trajectory
- Explicit driving function for sources under uniform motion
- Proof of implicit solution - explicit solution high frequency equivalency

Thesis group III-IV: SFS of moving sources

- Example: synthesis of harmonic moving source



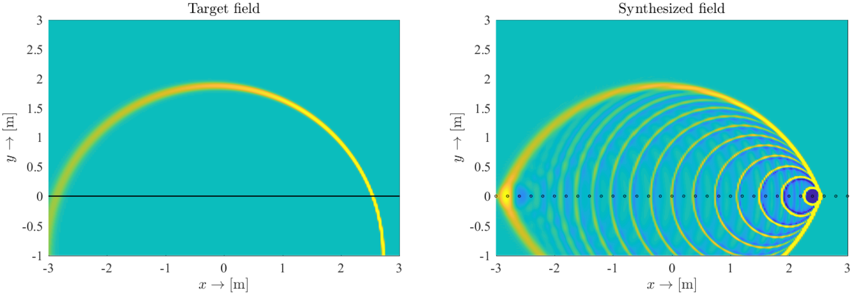
- Result:
  - phase correct synthesis over the listening region
  - amplitude correct synthesis over the reference curve

- For uniform motion (e.g. above): closed form driving functions are derived

Thesis group III-IV: SFS of moving sources

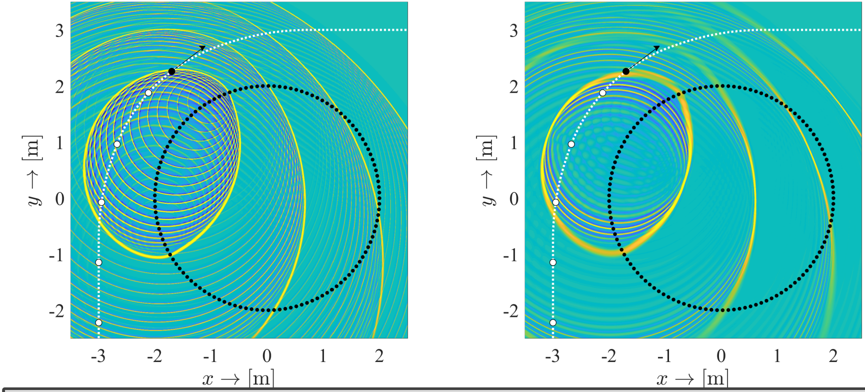
- Aliasing for moving sources:
  - aliasing artifacts are enhanced
  - frequency distortion is perceived: aliasing components suffer a different Doppler shift than the primary wavefront

    ↓
  - proper anti-aliasing is crucial



Thesis group III-IV: SFS of moving sources

Avoiding spatial aliasing



- Anti-aliasing criterion: by time-varying filtering

$$D(\mathbf{x}_0, \omega) = 0, \qquad \omega(\mathbf{x}_0, t) \geq \frac{\pi}{\Delta x} \frac{c}{|\hat{k}_t^P(\mathbf{x}_0, t)|}$$

- Circular SSD: optimal in the aspect of aliasing and referencing the synthesis

Thesis group III: Wave Field Synthesis of moving point sources

Thesis group summary:

- 3D Wave Field Synthesis for moving sources on arbitrary trajectory and excitation signal [J1, J3]
- 2.5D Wave Field Synthesis for moving sources on arbitrary trajectory and excitation signal, obtained by adapting the stationary phase approximation for time variant field [J3]
- Closed form 2.5D WFS driving functions for sources under uniform motion (↔ arbitrary case: propagation time delay must be expressed apriori) [J1]
- Frequency-domain 2.5D WFS driving functions for sources under uniform motion, derived directly in the frequency domain [J6]

Thesis group IV: Synthesis of moving sources in the wavenumber domain

Thesis group summary:

- 2.5D SDM driving functions for sources under uniform motion [J6]
- Proof, for frequency-domain 2.5D WFS and 2.5 SDM driving functions coincide (similarly to the static case) [J6]
- Analytical treatment of spatial aliasing artifacts regarding frequency distortion artifacts and optimal SSD shape choice [C8]
- Extension of the anti-aliasing strategy for the case of moving sources [C11]

Summary

- Main results
  - Generalized Wave Field Synthesis framework for arbitrary virtual fields and SSD shapes
  - Spatial form of the explicit approach, proven to coincide with the generalized WFS solution
  - 2.5D WFS and explicit driving function for sources, moving on arbitrary trajectories
  - Anti-aliasing strategy for both static and dynamic virtual fields
- Outlook
  - Discussion of focused virtual sources
  - Efficient implementation of the results

Journal papers

- [J1] G. Firtha, P. Fiala. ”Wave Field Synthesis of Moving Sources with Retarded Stationary Phase Approximation” In: *JAES* 63.12 (2016). IF: 0.774, C: 3
- [J2] G. Firtha, P. Fiala, F. Schultz, S. Spors.“Improved Referencing Schemes for 2.5D Wave Field Synthesis Driving Functions.” In: *IEEE/ACM TASLP* (2017). IF: 2.95, C: 3, pp. 1117–1127.
- [J3] G. Firtha, P. Fiala. ”Wave Field Synthesis of moving sources with arbitrary trajectory and velocity profile” In: *JASA* (2017). IF: 1.572, C: 3
- [J4] G. Firtha, P. Fiala, F. Schultz, S. Spors. ”On the General Relation of Wave Field Synthesis and Spectral Division Method for Linear Arrays” In: *IEEE/ACM TASLP* (2018). IF: 2.95
- [J5] F. Winter, F. Schultz, G. Firtha, S. Spors. ”A Geometric Model for Prediction of Spatial Aliasing in 2.5D Sound Field Synthesis” In: *IEEE/ACM TASLP* (2019). IF: 2.95.
- [J6] G. Firtha, P. Fiala. ”Sound Field Synthesis of Uniformly Moving Virtual Monopoles” In: *JAES* (2015). IF: 0.774, C: 5

Thank you for the attention!

Reviewer's questions

It would be really interesting to add a little bit on the validity of the high-frequency approximation that you are making. It would be interesting to see a case where it does not hold.

Reviewer's questions

Validity of high-frequency / far- field approximation (local plane wave approximations) depends on

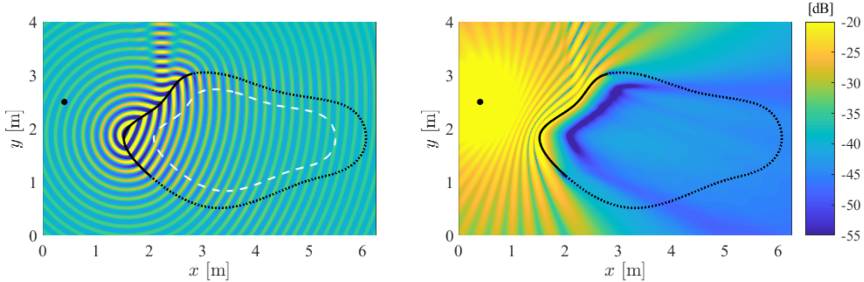
- SSD shape: validity of Kirchhoff approximation

$$\left.\begin{array}{l}\bullet\ \text{virtual source model / position}\\ \bullet\ \text{reference curve position}\end{array}\right\}\ \text{validity of the SPA}$$

Reviewer's questions

Dependence on SSD shape and source frequency

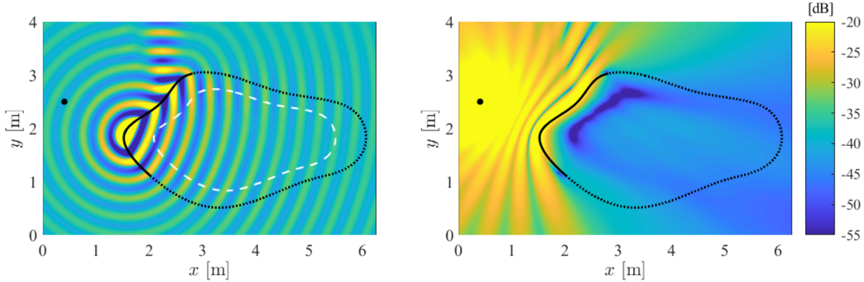
- Virtual source model: 3D point source
- $f_0 = 2\ \mathrm{kHz}, \quad \mathbf{x}_\mathrm{s} = [0.4\ \mathrm{m},\ 2.5\ \mathrm{m}]$



Reviewer's questions

Dependence on SSD shape and source frequency

- Virtual source model: 3D point source
- $f_0 = 1\ \mathrm{kHz}, \quad \mathbf{x}_\mathrm{s} = [0.4\ \mathrm{m},\ 2.5\ \mathrm{m}]$



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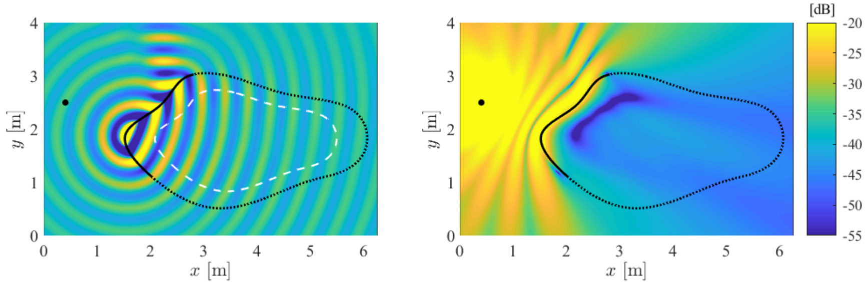

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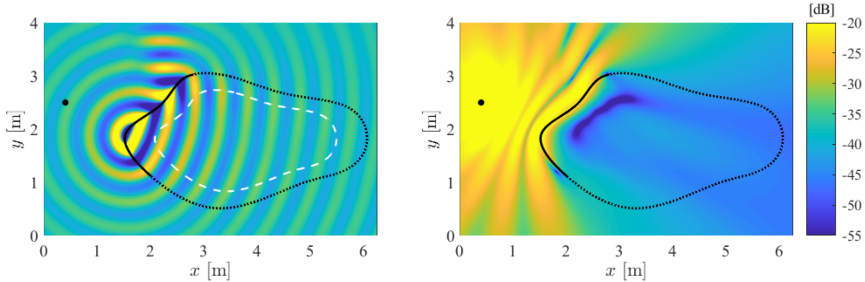

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Dependence on SSD shape and source frequency

- Virtual source model: 3D point source
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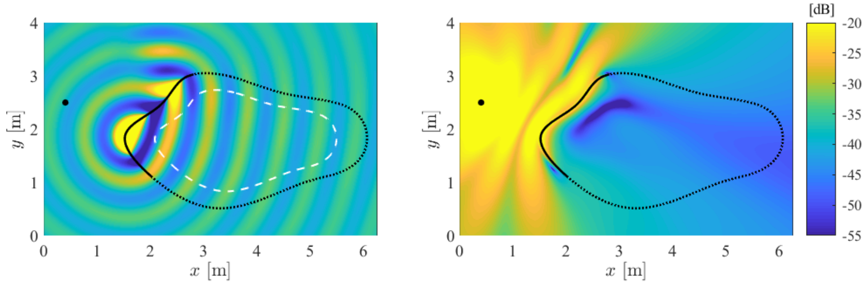

Reviewer's questions

Dependence on SSD shape and source frequency

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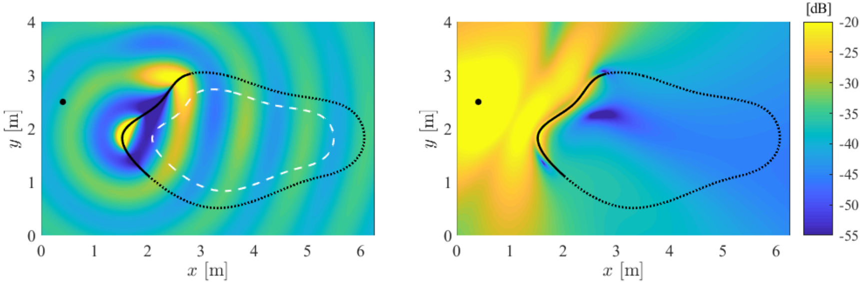

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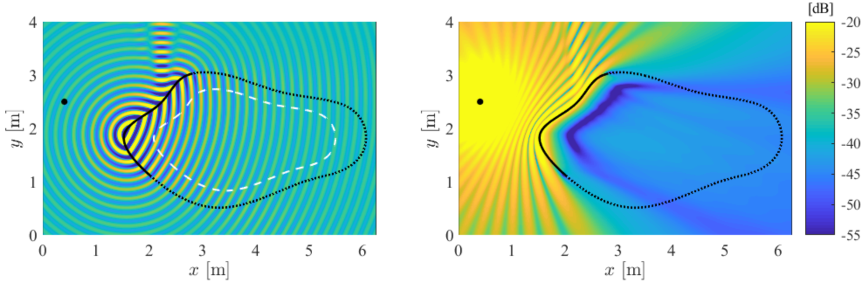

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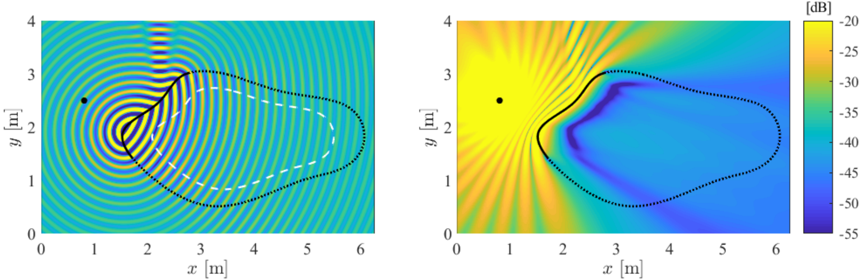

Reviewer's questions

Dependence on SSD shape and source frequency

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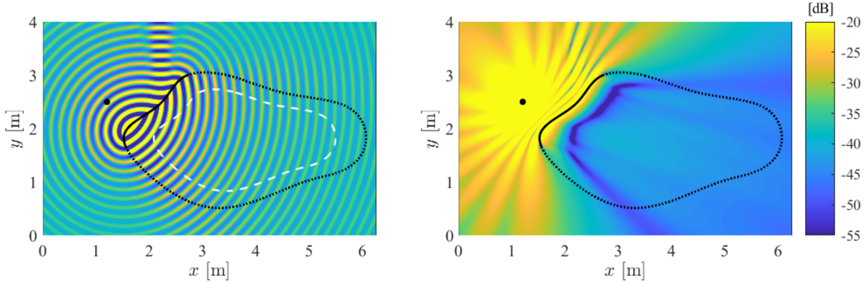

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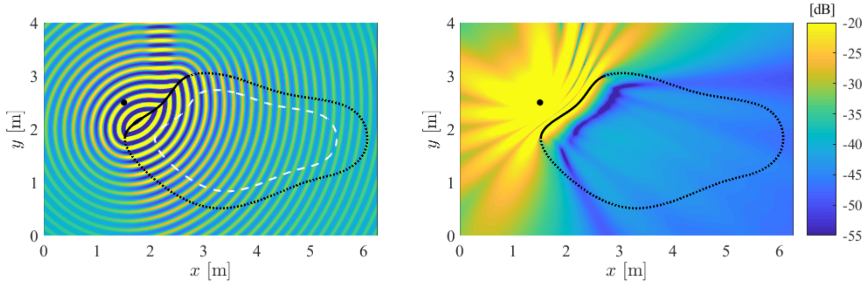

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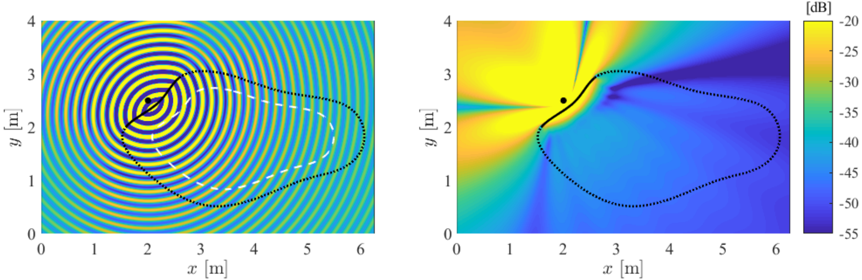
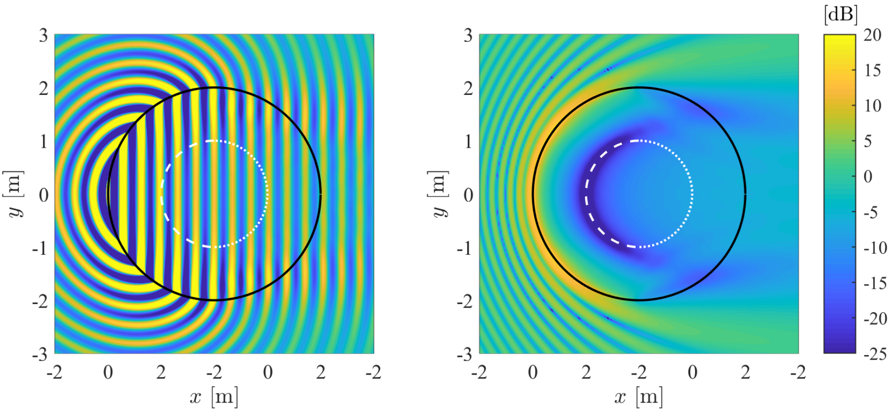


Reviewer's questions

Dependence on source frequency

- Virtual source model: Plane
- $f_0 = 1$ kHz



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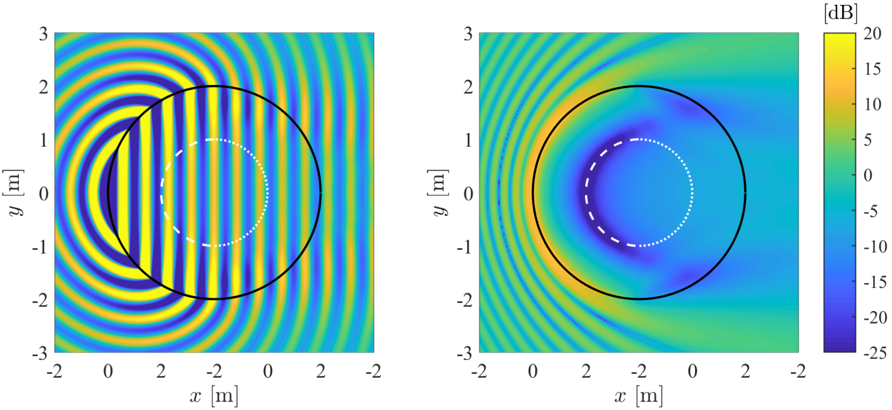

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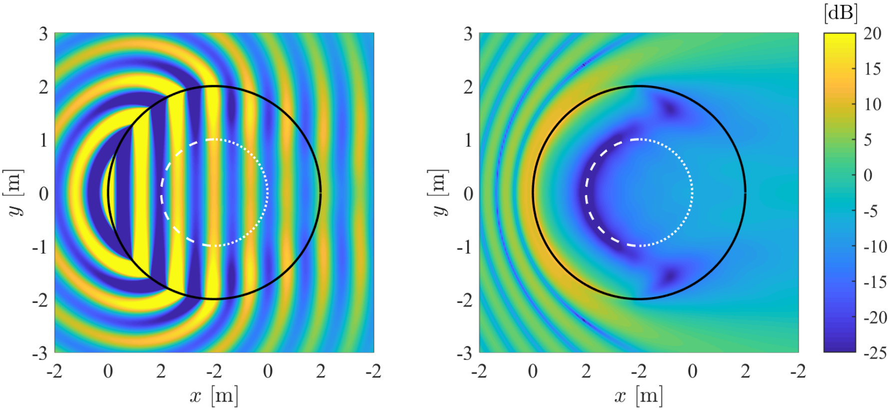

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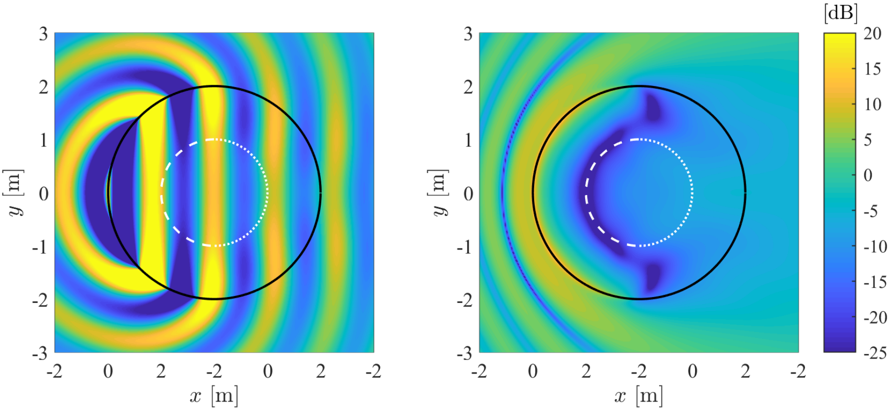
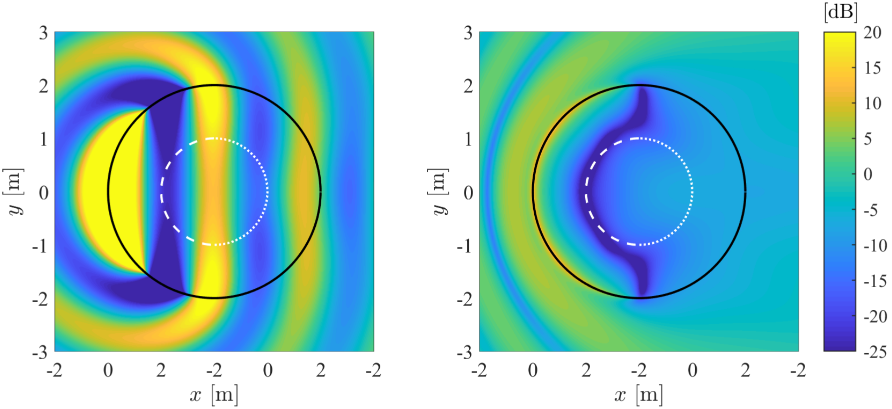


Reviewer's questions

Dependence on source frequency

- Virtual source model: 3D point source
- $f_0 = 0.2\ \mathrm{kHz}, \quad \mathbf{x}_\mathrm{s} = [0.4\ \mathrm{m},\ 2.5\ \mathrm{m}]$



Reviewer's questions

Dependence on source frequency

- Virtual source model: 3D point source
- $f_0 = 0.1\ \mathrm{kHz}, \quad \mathbf{x}_\mathrm{s} = [0.4\ \mathrm{m},\ 2.5\ \mathrm{m}]$