Sound Field Reproduction Applying Stochastic Secondary Sources

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1. Introduction

Spatial sound reproduction systems with a large number of densely spaced loudspeakers have been the target of research for over a decade. The aim of these systems is to physically synthesize a desired sound field in an extended listening area. Wave field synthesis (WFS) [1–4] and spectral division method (SDM) [5] are two well known analytical methods for sound field synthesis.

Traditional methods, like wave field synthesis involve identical acoustical point sources as secondary sources for the reproduction of the desired sound field. The general theory of spectral division method gives us the possibility to derive an analytical formulation to the case, where directional secondary sound sources are employed [6], as long as we suggest identical secondary sources. However, the requirement of applying linear, continuous distribution of identical point sources is not realizable in practice. Typically sound field reproduction is realized by using dynamic loudspeakers arranged along a line. Obviously, real-life loudspeakers – even from the same type – are not identical; both their temporal and spatial characteristics are different. To the author's knowledge the effects of these stochastic properties have not been investigated in details yet. The objective of the research is to give a rigorous mathematical description of the radiated noise and to examine if these stochastic artifacts may have an audible effect on the synthesized sound field.

First, we present a model and a general formula that describes the effect of several different stochastic sources. From all of these effects, the paper focuses stochastic loudspeaker sensitivity.

2. Model for stochastic sound reproduction

General theory

The general problem arrangement can be seen in figure 1. The generated wave field is written as the sum of the wave fields of individual monopoles, called *secondary sources*, that form the *secondary source distribution*. Our aim is to find the secondary sources' *driving function* that results in a generated field equal to that of the *virtual source*. The term 2.5-dimensional synthesis refers to the fact, that instead of a planar secondary source distribution – that would be needed to perfectly synthesize the sound field in the listener's half space – we employ a linear distribution of three-dimensional point sources. In this configuration perfect reproduction is only possible on the *reference line* that is parallel to the secondary source line. In other locations of the *synthesis plane* – a horizontal plane containing the secondary source line – amplitude errors occur.

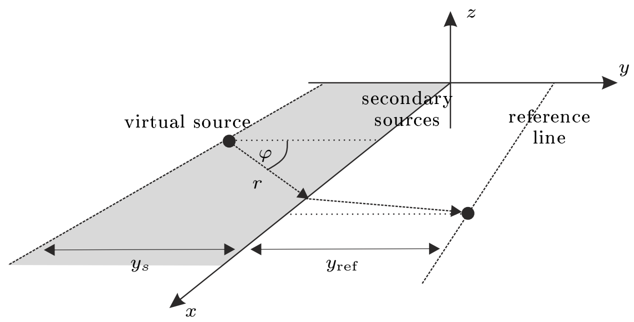

Figure 1: Geometry for the derivation of 2.5-dimensional synthesis operator in spatial and wave number domain

The resulting sound field, generated by the secondary source distribution on the line $[x\ y\ 0]^{\mathrm{T}}$ can be written as a convolution form:

$$P(x,y,\omega) = \int_{-\infty}^{\infty} Q(x-x_0,\omega) G(x_0,y,\omega) \mathrm{d}x_0, \qquad (1)$$

where $Q(x,\omega)$ is the driving function, while $G(x,y,\omega)$ is the three-dimensional free-field Green's function. In the $z=0$ plane it is given by

$$G(x,y,\omega) = \frac{1}{4\pi} \frac{\mathrm{e}^{-\mathrm{j}k\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}. \qquad (2)$$

For the sake of brevity the notation of frequency dependence will be further omitted.

Traditional WFS driving function formulation was given by Verheijen [1]. With the notation shown in figure 1 and for a virtual monopole at $\mathbf{x}_{\mathrm{s}} = [x_s\ (-y_s)\ 0]^{\mathrm{T}}$ the driving function takes the form

$$Q_{\mathrm{WFS}}(x) = \sqrt{\frac{\mathrm{j}k}{2\pi}} \sqrt{\frac{y_{\mathrm{ref}} y_s^2}{y_{\mathrm{ref}} + y_{\mathrm{s}}}} \frac{\mathrm{e}^{-\mathrm{j}kr}}{r^{\frac{3}{2}}}. \qquad (3)$$

For the sake of simplicity the origin of the coordinate system is chosen so that the virtual monopole is located at $x_s = 0$.

As it was pointed out in [5], the convolution form of the Rayleigh integral (1) can be transformed into a spectral multiplication, thus, the spectrum of the driving function can be written as:

$$\tilde{Q}_{\mathrm{SDM}}(k_x) = \frac{\tilde{P}(k_x, y_{\mathrm{ref}})}{\tilde{G}(k_x, y_{\mathrm{ref}})}, \qquad (4)$$

where $\tilde{P}(k_x, y_{\mathrm{ref}})$ and $\tilde{G}(k_x, y_{\mathrm{ref}})$ are the spectra of the sound field of the virtual source and Green's function at the origin, measured on the reference line. This is the concept of the SDM method.

In the author's previous work it was shown that for a virtual monopole under certain assumption the driving functions derived by WFS and SDM can be considered to be equal [7]. As these approximations hold for the particular case, presented in the paper, the driving function $Q(x)$ may refer to a driving function derived by any of these methods.

Modelling extended sources

In this treatise a similar treatment of extended sound sources is used as it was given by [1]. We assume that the extended source – the loudspeaker diaphragm – is located at the $y = 0$ plane, vibrating with a normal surface velocity $V_{\mathrm{n}}(x, 0, z)$. The radiated field in the plane $z = 0$ can be written analytically utilizing the Rayleigh I integral theorem, again in a convolutional form:

$$P(x, y, 0) = -\frac{\mathrm{j}\rho_0 ck}{2\pi} V_{\mathrm{n}}(x, 0, z) * *G(x, y, z) \quad (5)$$

If the vertical size of the speaker is small compared to the wavelength, then the radiated sound field can be written as a one-dimensional convolution along the x-axis:

$$P(x, y, 0) \approx G(x, y, 0) * h(x), \quad (6)$$

where $h(x)$ is a low-frequency farfield approximation of the surface velocity function:

$$h(x) = -\frac{\mathrm{j}\rho_0 ck}{2\pi} \int_{-\infty}^{\infty} V_n(x, 0, z)\mathrm{d}z. \quad (7)$$

From here the one-dimensional function $h(x)$ will be termed as spatial extension function or strength function. Numerical simulations showed that this approximation holds well to 2-3 kHz frequency range besides about 0.1-0.2 m diaphragm diameter. Using the approximation the reconstructed sound field of a virtual monopole can be written as

$$P(x, y) = Q(x) * h(x) * G(x, y, 0). \quad (8)$$

Note, that here the discretization of the secondary source distribution is omitted. It could be treated analytically by sampling the $Q(x)$ driving function before the convolution.

To examine, how the extension function varies as the function of diaphragm geometry and frequency a mechanical finite element model was created. Simulation was carried out for a simple loudspeaker model with a polypropylene diaphragm, driven with the force exerted by the voice coil. The membrane parameters, material properties and dimensions were taken from related research [8, 9].

The simulation results were used to approximate the extension function $h(x)$ by evaluating equation (7) at 1 kHz. The resulting function is shown in figure 2.

Modelling extended stochastic sources

So far we considered sound sources with deterministic properties. Obviously, real-life speakers always have slightly different characteristics, even if they are of the same type. In the present work we take two stochastic components into consideration:

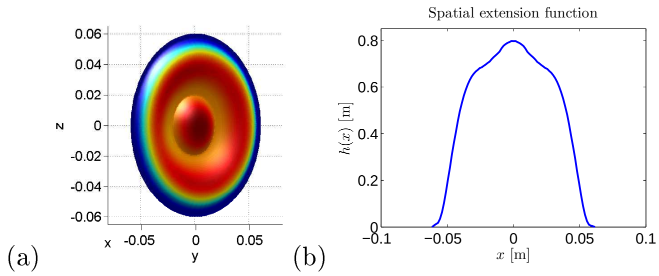

Figure 2: (a) Excursion amplitude of the FEM loudspeaker model and (b) spatial extension function, calculated numerically at 1 kHz

- *Stochastic sensitivity* models that each loudspeakers have slightly different electro-mechanical transducer gain, depending on electrical, magnetic and mechanical properties. The sensitivity noise can be modeled as an additive noise $n_{\mathrm{s}}(x)$ to the nominal sensitivity. If the sensitivity is normalized, then the resulting noisy driving function can be written as:

  $$Q_n(x) = Q(x)(1 + n_{\mathrm{s}}(x)) = Q(x) + Q(x)n_{\mathrm{s}}(x), \quad (9)$$

  The result shows that sensitivity noise acts as a multiplicative noise on the driving function.

- *Mechanical anisotropy* is introduced to model the stochastic effects originating from surface rugosity, space dependent material properties and modal behavior. As these components will directly affect the surface velocity function, they can be treated as an additive noise $n_{\mathrm{m}}(x, \tau)$ to the extension function $h(x)$:

  $$h_n(x) = h(x) + \sup(h(x)) \left(n_{\mathrm{m}}(x, \tau)\right). \quad (10)$$

  where dependency on variable τ indicates the different realizations on different source elements.

Substitution of the noisy driving function and extension function into equation (8) reveals that the radiated field can be written as the sum of four components:

- the ideal noiseless sound field of the virtual source
- radiated noise resulting from stochastic sensitivity
- radiated noise resulting from the mechanical noise
- a combined effect of the two noise components

The final aim of the research is the analytical description of all noise components. In the present paper from now we focus our investigation on the effects of stochastic sensitivity.

3. Investigation of the effects of stochastic sensitivity

In the current section we investigate the properties of the radiated noise for the case of a synthesized virtual monopole.

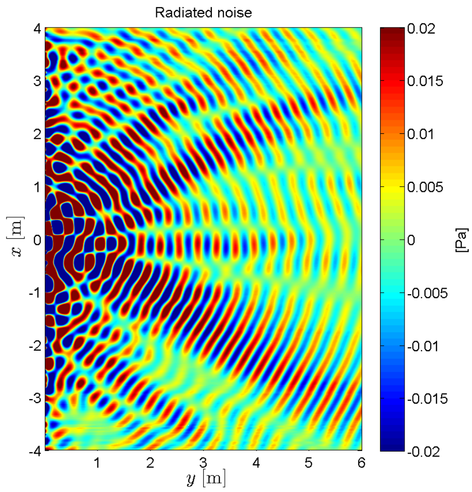

Figure 3: Real value of the radiated noise: $\mathrm{Re}\{N(x,y)\}$

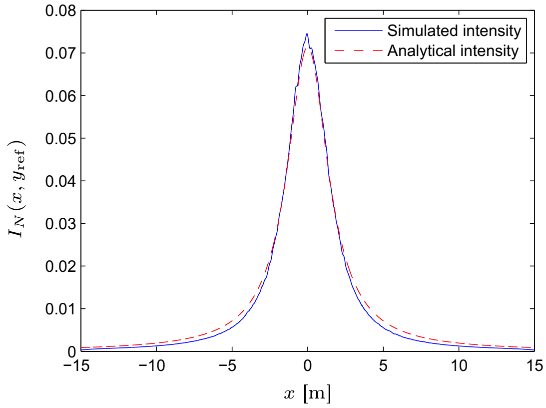

Figure 4: Comparison of analytical and simulated intensity $I_N(x,y)$ on reference line

As most physical random processes, the sensitivity noise is influenced by many random parameters, thus its distribution can be considered to be Gaussian: $n_{\mathrm{s}}(x) \in \mathcal{N}(0,\sigma_{\mathrm{s}}^2)$. It is obvious, that the sensitivity noise is uncorrelated, since there's no dependency between the sensitivity of different source elements. The autocorrelation of this component is therefore $R_{n_{\mathrm{s}}n_{\mathrm{s}}}(\tau) = \sigma_{\mathrm{s}}^2\delta(\tau)$. It can be easily proven that the resulting multiplicative noise component $n'_{\mathrm{s}}(x) = Q(x)n_{\mathrm{s}}(x)$ (from equation (9)) is a *non-stationary* white noise with the autocorrelation function $R_{n'_{\mathrm{s}}n'_{\mathrm{s}}}(x,\tau) = |Q(x)|^2\delta(\tau)$.

Autocorrelation function of the radiated noise

From equation (8), the radiated noise, originating from the stochastic sensitivity can be written as:

$$N(x,y) = n'_{\mathrm{s}}(x) * (h(x) * G(x,|y|)) \qquad (11)$$

In the frequency range of interest (f < 1.5 kHz) it is feasible to assume that the support of the source extension function is smaller than the wavelength of the Green's function. In this case as a farfield approximation the frequency shaping effect of the linear filtering may be omitted, therefore

$$h(x) * G(x,y) \approx \mathrm{E}_h G(x,y), \qquad (12)$$

where $\mathrm{E}_h = \int_{-\infty}^{\infty} h(x)\mathrm{d}x$. Numerical simulations, carried out utilizing the FEM loudspeaker model also confirmed the validity of this approximation.

Applying the approximation, the autocorrelation function of the radiated noise can be written by definition:

$$R_{NN}(x,y,\xi,\nu) = \sigma_s^2\mathrm{E}_h^2 \int_{-\infty}^{\infty} G(x-\tau,y) \overline{G}(x-\tau-\xi,y-\nu)|Q(\tau)|^2\mathrm{d}\tau. \qquad (13)$$

Due to the complexity of the expression the analytic investigation has not been carried out yet.

In figure 3 the real value of the radiated noise can be seen, which simulation was carried out by evaluating equation (11) over the whole listening area for a virtual source, located 1 m behind the secondary array, radiating at 1 kHz. The simulation result suggests that there's a strong radial correlation into the direction of a virtual sound source, however studying maximum correlation direction of the field described by equation (13) is the topic of a future research. The exact position of this virtual source is studied in the following subsection.

Intensity of radiated noise

Besides the autocorrelation function the intensity distribution function is an important – maybe more informative – property of the radiated noise as it is comparable to the energy of the ideal, noiseless synthesized field.

The intensity function of the radiated noise from equation (13) can be written as

$$I_N(x,y) = R_{NN}(x,y,0,0) = \sigma_s^2\mathrm{E}_h^2|G(x,y)|^2 * |Q(x)|^2. \qquad (14)$$

The convolution can be carried out by means of farfield approximations in spectral domain. Here without derivation, the resulting intensity function is found to take the form:

$$I_N(x,y) = \sigma_s^2\mathrm{E}_h^2 \frac{\frac{2}{3}y_s + y}{y_s + y_{\mathrm{ref}}} \frac{y_{\mathrm{ref}}}{y} \frac{k/\pi}{r'^2}. \qquad (15)$$

where $r' = \sqrt{x^2 + (y_{\mathrm{ref}} + \frac{2}{3}y_s)^2}$. The expression is even simpler on the reference line:

$$I_N(x,y_{\mathrm{ref}}) = \sigma_s^2\mathrm{E}_h^2 \left(1 - \frac{\frac{1}{3}y_s}{y_s + y_{\mathrm{ref}}}\right) \frac{k/\pi}{r'^2}. \qquad (16)$$

Figure 4 compares the above result with an appropriate Monte Carlo simulation. It can be stated that there is a good match between simulated and analytical average intensity over a large parameter range.

By investigating the analytical formulation of intensity it can be noted that r' gives the distance between points of the reference line and a point source, located at $(0, -\frac{2}{3}y_s, 0)$, therefore, it is closer to the secondary source distribution than the original virtual point source, located at $(0, -y_s, 0)$. In other words the equation states that *the intensity of the noise, radiated from the secondary sources due to the stochastic sensitivity fluctuates around the intensity of the sound field, generated* *by a phantom source, located closer to the secondary source distribution by a factor of $\frac{1}{3}$, radiating on the same frequency as the virtual source.* The rate of oncoming of the virtual source to the secondary source line is specified by the driving function, thus the present approaching originates from the 2.5-dimensional driving function correction.

We may define the relative noise intensity (or noise-to-signal ratio) as the ratio of noise intensity and intensity of the field of the virtual source. The driving function was derived so that the synthesis is perfect on the reference line. Applying the same approximations as before and utilizing interchangeability of extensions of the secondary source and the virtual source [7], the field of the virtual source on the reference line reads $P(x, y_{\mathrm{ref}}) = \mathrm{E}_h \frac{\mathrm{e}^{-\mathrm{j}kr}}{r}$. From this, the noise-to-signal ratio is expressed as

$$I_{\mathrm{rel}}(x) = 10\lg\sigma_s^2 \left(\frac{\frac{2}{3}y_s + y_{\mathrm{ref}}}{y_s + y_{\mathrm{ref}}} \right) \frac{k}{\pi} \left(\frac{r}{r'} \right)^2, \qquad (17)$$

where $r = \sqrt{x^2 + (y_s + y_{\mathrm{ref}})^2}$. The result states that the relative noise intensity is the largest in front of the virtual sound source ($x = 0$), while in lateral positions the noise intensity is approximately proportional to the target sound field. The maximum of the relative noise intensity is given by:

$$I_{\mathrm{rel,max}} = 10\lg\sigma_s^2 \left(\frac{y_s + y_{\mathrm{ref}}}{\frac{2}{3}y_s + y_{\mathrm{ref}}} \right) \frac{k}{\pi} \qquad (18)$$

As an example let's investigate the synthesis of the sound field of a point source radiating at 1 kHz. For the sake of a real-life example the standard deviation from the nominal sensitivity was measured for Behringer Truth B2030A studio monitors, for which the result was $\sigma_s = 7$ %. As the term, containing y_s and y_{ref} varies in the $[1, \frac{3}{2}]$ interval, the maximal relative intensity of the phantom source will be between -15 and -13 dB. The audibility of such a phantom source in the presence of the ideal synthesized sound field of the original virtual source is the topic of a further research.

4. Conclusion

In the present contribution a treatise on stochastic sound field reproduction was given. In the presented model spatially extended source elements were considered as secondary source elements. Extending the traditional model we also took the stochastic properties of real-life sound sources into consideration. From all of the modeled noise components, stochastic loudspeaker sensitivity was examined in details.

For the intensity of the radiated noise – by simplifying the general autocorrelation formula at zero space lag – a well-usable analytic formula can be given. It was shown, that the relative noise intensity – compared to the field of the noiseless virtual source – is proportional to the energy of the sensitivity noise and to the examined frequency. The main finding of the research was that the radiated noise seems to originate from a point source with a stochastic directivity, slightly closer to the secondary source distribution that the original virtual source. The approaching factor $\frac{2}{3}$ is a direct consequence of the 2.5-dimensional correction of the WFS driving function. The virtual source position mismatch due to driving functions belonging to different secondary source distribution is still an open question.

In the present proceeding only a part of the research treating stochastic sound field synthesis was presented. Giving a mathematical formulation for the mechanical noise and the evaluation of the combined effect of sensitivity noise and mechanical noise is the topic of future work. Besides this, the sensitivity noise can be considered frequency dependent. To investigate the effects of the sensitivity noise in case of synthesizing wave fronts emitted by virtual sources time domain analysis is needed. This aspect of examination is also yet to be carried out.

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