Synthesis of a moving virtual sources with Wave Field Synthesis

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Introduction

Wave Field Synthesis is one of the most prominent Sound Field Synthesis (SFS) techniques, aiming to physically reproduce an arbitrary sound field, most often generated by a virtual point source or a plane wave. To achieve this, a densely spaced loudspeaker array, termed as *secondary source distribution* (SSD) is driven by a *driving function* derived either in spatial or spectral domain. Traditional WFS technique yields the driving functions for a linear SSD from the Rayleigh integral formulation of the target field [1, 2, 3].

Besides the reproduction of stationary sources the synthesis of moving sound sources has gained increasing interest, having a straightforward application in virtual reality systems or in cinemas. In these cases the primary objective is the correct reproduction of the Doppler-frequency shift [4].

Early implementations simulated the source motion as a sequence of stationary positions, which approach ends up in a Doppler-like effect with several undesired artifacts [4, 5]. In recent studies efforts were made to give analytically correct driving functions by incorporating the dynamics of a uniformly moving point source [6]. In a former paper by the authors a mathematically consistent solution was derived for both WFS and an SFS technique providing the analytical reference solution, termed as the Spectral Division Method [7]. The necessity of numerical inverse Fourier-transforms – thus the great computational complexity –, however, made the solution infeasible, when applied for sources with arbitrary temporal excitation signal.

The present contribution adapts the traditional WFS theory to uniformly moving virtual sound sources. By using the same assumptions as for the stationary case analytically correct driving functions are derived both in the frequency and the temporal domain. The applicability of the derived driving functions are demonstrated via numerical simulation examples.

Synthesis of a moving source

The general WFS arrangement is presented in Figure 1. It is assumed that the SSD is a linear, continuous, identical point source distribution, located along the x-axis at $[x,\ 0,\ 0]^{\mathrm{T}}$. The virtual sound source is a point source, traveling behind the SSD with a uniform velocity v at an arbitrary angle of inclination to the x-axis (φ), and located at $[x_s\ -y_s\ 0]^{\mathrm{T}}$ at the time origin. Our aim is to find the SSD driving function $d(x_0, t_0)$, so that the weighted sum of the SSD elements' individual sound fields equals

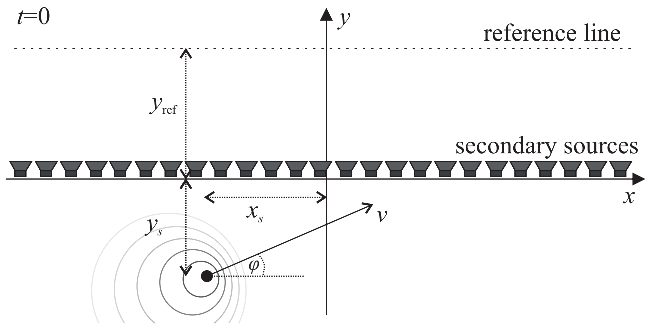

Figure 1: WFS geometry under discussion.

to that of the moving source on the reference line:

$$p(x, y_{\mathrm{ref}}, 0, t)_{\mathrm{moving}} = \iint_{-\infty}^{\infty} d(x_0, t_0) h(x - x_0, y_{\mathrm{ref}}, t - t_0) \mathrm{d}x_0 \mathrm{d}t_0, \quad (1)$$

where $h(\mathbf{x}, t)$ is the impulse response of a secondary source element.

The WFS solution for the derivation of the driving functions relies on the 2.5-dimensional formulation of the Rayleigh-integral [1]. The Rayleigh-integral states that by driving a planar source distribution with the normal derivative of the desired sound field taken on the secondary source plane would ensure a perfect reconstruction in front of the plane. The Rayleigh-integral can be therefore applied directly to the formulation of the moving source's sound field.

Our derivation follows the traditional WFS derivation. In the followings only major steps are exposed.

- The analytical description the sound field, generated by a sound source moving parallel to the x-axis can be found in the related literature (e.g. [6, 9]). A simple extension of the formulation for sources moving in an arbitrary direction can be found in [7].
- In order to carry out the stationary phase approximation, a moving harmonic source is assumed, oscillating at an angular-frequency ω_0.
- The normal derivative of the target sound field can be expressed analytically. For simplicity high-frequency approximations are applied (see [2] for the stationary situation). At this point the 3D WFS driving functions for planar SSDs are obtained explicitly
- The normal derivative of the target sound field is applied to the Rayleigh-integral, and integration along

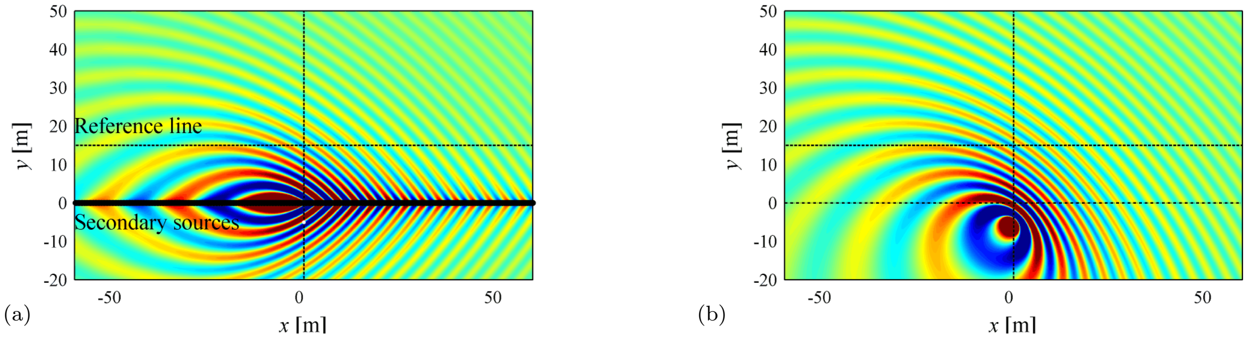

Figure 2: Snapshot of the synthesized field (a) and the reference field (b) at $t = 0$.

the z-dimension can be carried out analytically by using the stationary phase approximation

- From the remaining line-integral the spatio-temporal Green's function can be deconvolved analytically
- In order to restrict perfect synthesis on a reference line instead of a reference point a further stationary phase approximation is applied along x-dimension (see [1] for the stationary situation)

After carrying out the procedure described above, the driving function for a moving harmonic point source is obtained as

$$d(x_0, t_0, \omega_0) = -\sqrt{\frac{\mathrm{j}\omega_0}{2\pi c}}\sqrt{\frac{y_{\mathrm{ref}}}{y_{\mathrm{ref}} - y_{\mathrm{e}}(x_0, t_0)}} \; y_{\mathrm{e}}(x_0, t_0)\frac{\mathrm{e}^{\mathrm{j}\omega_0(t_0 - \tau(\mathbf{x}', t_0))}}{\Delta(\mathbf{x}', t_0)^{\frac{3}{2}}}, \quad (2)$$

with the coordinates $\mathbf{x}' = [x',\ y',\ z']^{\mathrm{T}}$ given by

$$x' = \cos\varphi(x_0 - x_s) - \sin\varphi y_s \quad (3)$$
$$y' = -\sin\varphi(x_0 - x_s) - \cos\varphi y_s \quad (4)$$
$$z' = 0, \quad (5)$$

and $y_{\mathrm{e}}(x_0, t_0)$ being the y-coordinate of the moving source at the time instant of emission. The attenuation factor and the propagation time delay (between measurement time t_0 and the time of emission) are given as

$$\Delta(\mathbf{x}, t) = \sqrt{(x - vt)^2 + (y^2 + z^2)(1 - M^2)}, \quad (6)$$

and

$$\tau(\mathbf{x}, t) = \frac{M(x - vt) + \Delta(\mathbf{x}, t)}{c(1 - M^2)}, \quad (7)$$

where $M = v/c$ is the *Mach number*. Finally the virtual source position at the time instant of emission can be calculated as

$$y_{\mathrm{e}}(x_0, t_0) = y_s + v\sin\varphi(t_0 - \tau(\mathbf{x}', t_0)). \quad (8)$$

One can notice, that the resulting driving functions take the same form, as the traditional WFS driving functions for a stationary virtual source, as given e.g. [2]. By substituting $v = 0$ into equation (2) one obtains the traditional WFS driving functions, therefore the presented results can be regarded as a generalized WFS solution, valid both for moving and stationary virtual sources.

Just as for traditional WFS the driving functions can be inverse Fourier-transformed analytically, yielding the time domain WFS driving functions for moving sources with arbitrary excitation:

$$d(x_0, t_0) = -2\pi\sqrt{\frac{y_{\mathrm{ref}}}{y_{\mathrm{ref}} - y_{\mathrm{e}}(x_0, t_0)}} \; y_{\mathrm{e}}(x, t_0)\frac{q(t_0 - \tau(\mathbf{x}', t_0)))}{\Delta(\mathbf{x}', t_0)^{\frac{3}{2}}} * h(t_0), \quad (9)$$

with $q(t)$ denoting the source excitation time history, and $h(t) = \mathcal{F}_{\omega}^{-1}\left(\frac{\mathrm{j}\omega_0}{2\pi c}\right)$ describing the WFS prefilters.

Due to the perfect analogy, the results indicate that practical implementation of the moving source driving functions faces the same challenges, as for the case of a stationary source: the proper WFS prefiltering and the application of fractional delays.

It should be noted here, that the derived driving functions are valid only for virtual sources, located behind the SSD. Sources traveling with a given inclination angle will always be located in front of the SSD for a certain amount of time. In this time interval—due to the symmetry of the Rayleigh integral—the $y < 0$ half-plane becomes the plane of correct synthesis, and perfect synthesis is restricted to the line $y = -y_{\mathrm{ref}}$. In order to avoid corrupted synthesis a simple strategy is to mute those secondary sources that would synthesize a virtual source located in front of the SSD.

Application examples

Synthesis of a harmonic source

In the following examples the spatial characteristics of the synthesized sound field are investigated in details using the derived driving functions. Ideal synthesis was assumed, approximated by using and SSD, truncated at $L = 250$ m, sampled at $\Delta x = 0.1$ m.

As the first example the synthesis of a moving harmonic monopole is presented. The virtual source is located behind the SSD at $\mathbf{x}_{\mathrm{s}} = [0,\ -5,\ 0]^{\mathrm{T}}$ at the time origin, travels with an angle of inclination of $\varphi = 45°$ to the

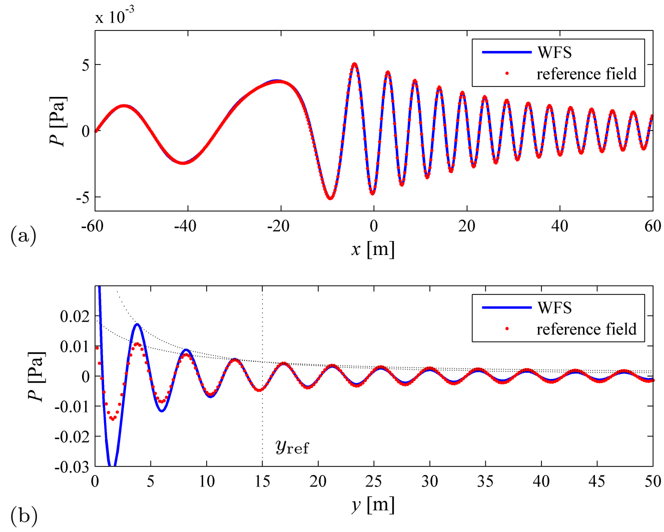

Figure 3: Comparison of the synthesized field and the reference along the reference line ($y = y_{\text{ref}}$) (a) and the y-axis ($x = 0$) (b).

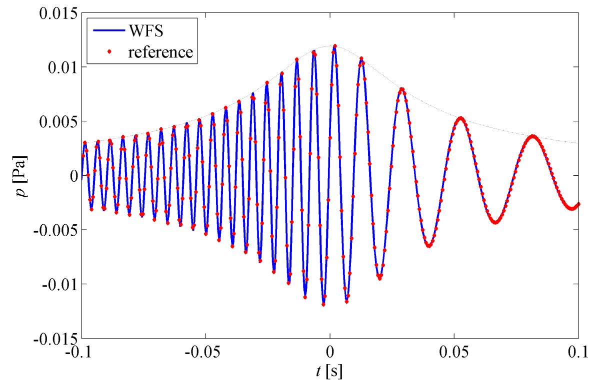

Figure 4: Time history of the SFS of a harmonic source, moving parallel to the SSD ($\varphi = 0$).

SSD with a velocity of $v = \frac{3}{4}c$, and radiates at an angular frequency of $\omega_0 = 2\pi \cdot 25$ rad/sec. The reference line is set to $y_{\text{ref}} = 15$ m. The driving functions are calculated by the evaluation of equation (2). The total duration of the simulated passby is 2 s with a sampling frequency of $f_s = 3$ kHz.

Figure 2 depicts snapshots from the synthesized and the analytical reference field, taken at $t = 0$. The presented snapshots indicate a phase-perfect synthesis in the whole $y > 0$ half-plane, i.e. in front of the secondary source distribution.

Examination of the spatial distribution of the synthesized field measured on the reference line (Figure 3 (a)) reveals that the presented driving functions ensure perfect synthesis regarding both amplitude and phase. Comparing the synthesized and the reference field along the y-axis it is confirmed, that amplitude correct synthesis is restricted to the reference line e.g. the amplitude of the fields coincide at $y = y_{\text{ref}}$. In other regions of the listening area amplitude errors are present. The characteristics of the amplitude deviation is completely analogous to the stationary case, which is investigated in details in e.g. [8].

Besides spatial characteristics, the proper reproduction of the virtual source's time history is of primary importance in the aspect of applicability. The second example for the synthesis of a harmonic source pass-by presents the time history of the synthesized field and the analytical reference, measured at $[0,\ y_{\text{ref}}, 0]^{\text{T}}$. Simulation results are depicted in Figure 4. In this case for the sake of transparency the source excitation frequency was increased to 50 Hz. In order to avoid a virtual source crossing the SSD –thus to simulate the entire pass-by with a single secondary array– the source moves parallel to the SSD. It can be seen that in this ideal case the synthesized field perfectly matches the analytical reference field both in amplitude and phase. This indicates that the presented WFS driving functions are capable of the perfect reconstruction of the Doppler frequency-shift.

Synthesis of a source with arbitrary excitation signal

As a more practical example, the synthesis of a moving source radiating with a wide-band excitation is investigated next. Again, an acoustic point source travels parallel to the SSD ($\varphi = 0$) with the velocity half of the speed of sound ($M = 0.5$), located at $\mathbf{x}_s = [0,\ -5,\ 0]^{\text{T}}$ in the time origin. The source emits a series of rectangular pulses, width of 5 ms and a time period of 20 ms (resulting in a source frequency of 50 Hz), sampled at $f_s = 10$ kHz. While evaluating equation (9), fractional delays and the WFS prefiltering characterized by $\sqrt{\omega}$ were implemented in the frequency domain, using filters with the same length as the input signal.

The simulation results are presented in Figure 5. The results indicate, that—aside from slight deviations due to the critical low-frequency WFS transfer, which arises from the high-frequency approximations applied—the presented solution ensures correct synthesis of a moving source with a wide-band excitation signal.

Conclusion

The contribution presented analytical WFS driving functions purely in the time domain for the synthesis of uniformly moving acoustic point sources.

The derivation followed the well-known traditional WFS derivation, adapted to the analytical description of a moving source. Applying the Rayleigh-integral theorem, the stationary phase approximation and an analytical Green's function deconvolution WFS driving functions were given for a harmonic point source, moving at arbitrary direction. Similarly to stationary WFS this formulation can be analytically inverse Fourier transformed to finally obtain time domain driving functions for the synthesis of sources with arbitrary excitation signals. In this way the presented approach overcomes the limitation of the authors' previous approach for the same problem, regarding its great computational complexity. The

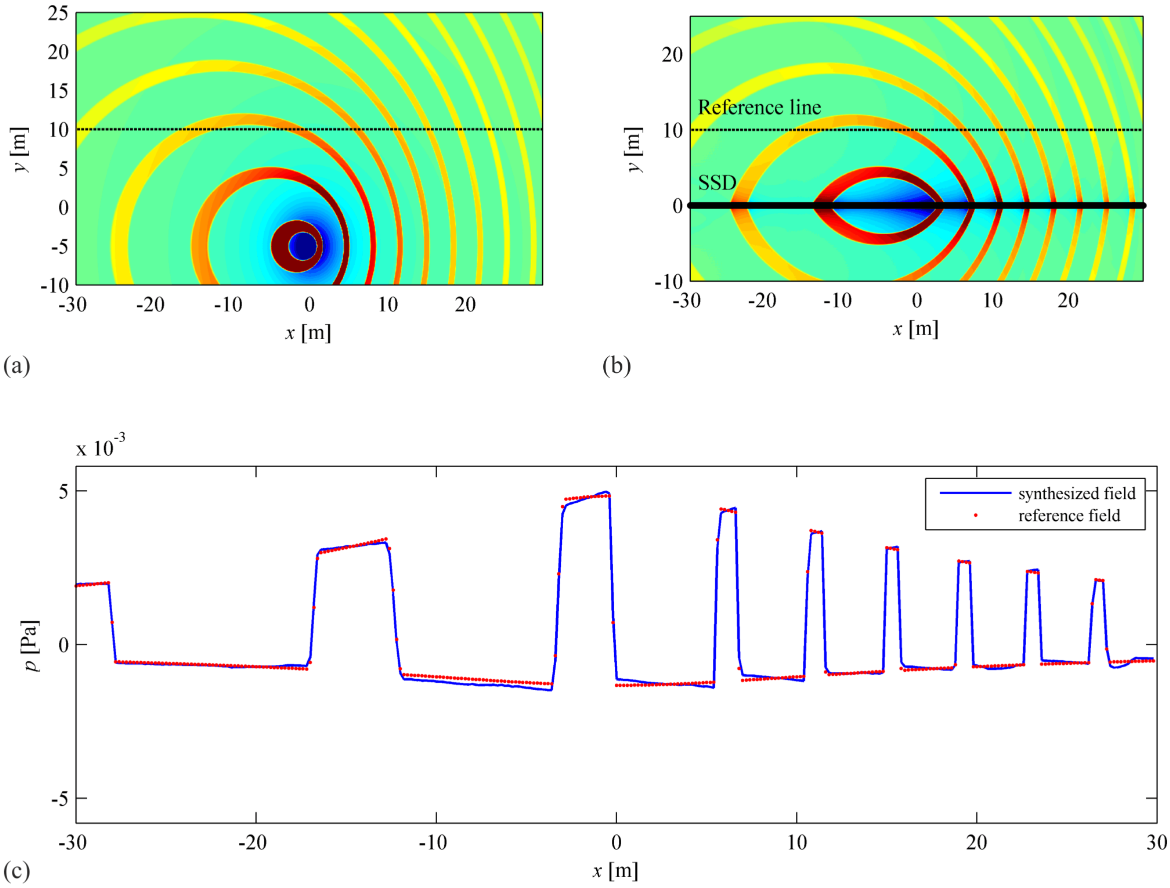

Figure 5: Synthesis of a moving source emitting a series of rectangular functions. Snapshot of the synthesized (a), the reference field (b) and the cross-section of fields taken on the reference line ($y = y_{\text{ref}}$) (c).

remaining implementation challenges –WFS prefiltering and fractional delays– are the well-known peculiarities of traditional WFS technique.

As an important result of the research it was proven, that by applying traditional approximations the resulting driving functions formally coincide with the driving functions for stationary sources, with the originally static distances changed to dynamic distances. The presented driving functions can be therefore regarded as generalized WFS driving functions, valid both for stationary and moving virtual sound sources. The validity of analytical results are demonstrated via numerical simulation examples, sources, moving at arbitrary direction radiating harmonically or the series of bandlimited pulses.

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