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NEW METHOD TO PREDICT APPLICABILITY LIMITS OF THE SABINE AND EYRING EQUATIONS BASED ON A LARGE NUMBER OF ROOMS

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ABSTRACT

The Sabine and Eyring equations for predicting the reverberation time are widely used in room acoustic design practice, often extending to spaces in which their applicability assumptions are not met. In this paper we test these widely used statistical equations along with the methods and applicability criteria of the European standard EN 12354-6:2003 on data set of more than 10,000 different rectangular room models and propose the application of new classification methods where the Sabine and Eyring equations can be used. Classification is performed in comparison with reverberation times predicted by numerical room acoustic simulations. We present the implementation of the method in the $soundy.ai$ application.

Keywords: *reverberation time, Sabine, Eyring, machine learning*

1. INTRODUCTION

1.1 Scope and motivation

Many of the current room acoustic design practice methods and requirements in national standards in different countries directly or indirectly rely on the Sabine formula for predicting the reverberation time, the required amount of absorption in a room, or noise levels. Despite the widespread application and studies in the literature, the diffuse space assumption is often difficult or ambiguous to be translated to practical room features, and reliable, clear empirical application limits with decision probabilities have not yet been published, leading to a high risk of false predictions, improper choice of methods, or sub-optimal design. Without these application limits, the acoustical consultant is left to rely on practical expertise to decide if a quick Sabine-based calculation is appropriate or more complex methods are needed. By more reliably predicting the applicability limits of statistical formulae, not only the calculations in the design phase would become more reliable, but the selection of design tools and methods, and consequently, the acoustical consultancy effort would also become more economical.

One of the main reasons for the lack of reliable empirical application limits of the Sabine [1] and Eyring [2] statistical prediction formulae is the lack of adequately large reference data sets.

This paper aims to revise the applicability limits of the EN 12354-6:2003 standard [3] of the Sabine and Eyring formulae in empty rectangular rooms.

We present a decision tree trained using machine learning methods that are based on certain room features related to geometrical and absorption characteristics to predict the applicability of the Sabine and Eyring equation at certain error bounds.

This decision tree is based on a simulated data set of nearly 30,000 rooms. The presented decision strategy improves the probability of a correct decision of applicability compared to the current standard by nearly 25 %.

2. METHODOLOGY

2.1 The classification model

We assume that the applicability of statistical formulae is a binary classification problem – for example, for any given room, the Sabine formula is either applicable or not. In this classification problem, inputs are certain properties or

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'features' of the room which are compared to a reference value, and a binary decision is made. Features and decisions based on them are ordered in a hierarchy and the output is a true or false value which, at a given probability is predicting whether the difference obtained by the statistical formulae and the reference ray tracing simulation is within given bounds.

Classification can be achieved by various methods such as logistic regression, k-nearest neighbors (k-NN), support vector machines (SVM), probabilistic classifiers (e.g., Bayes), decision trees, ensemble methods (e.g., random forest, gradient boosting machines) and deep learning architectures (convolutional neural networks or recurrent neural networks).

On the one hand, we consider our case a good fit for a supervised classification algorithm since our output is labeled. On the other hand, the possible large number of features and the relatively moderate amount of training data we have is still prone to overfitting issues.

Since we also aimed at creating a model capable of prediction of new rooms based on their features only, we choose to implement coarse decision trees and when an increased complexity is allowed we use bootstrap aggregating (bagged) trees.

Coarse decision trees are a type of decision tree algorithm that partitions the feature space into large rectangular regions, rather than recursively splitting the space into smaller and smaller regions. The algorithm is designed to be computationally efficient and suitable for high-dimensional feature spaces, where traditional decision tree algorithms can become impractical due to their exponential complexity. While they may not achieve the same level of accuracy as more complex algorithms, they can provide a good trade-off between accuracy and computational complexity.

On the other hand bootstrap aggregation is an ensemble learning method used to improve the performance and stability of decision tree models by combining multiple decision trees' predictions by taking a majority vote. In the bagging process bootstrapping is done by creating multiple datasets from the training data by randomly sampling the original dataset. Each bootstrap sample has the same size as the original dataset, but some instances may be repeated. A decision tree is created on each bootstrap sample and when making a prediction for a new data point, all trained decision trees are evaluated and the class label that receives the majority vote yields the final decision value. This bagged tree structure effectively reduces the variance of individual decision trees, improves accuracy and handles noisy data better. One possible implementation of bagged trees is the random forest algorithm which selects a random subset of features at each split during the tree-building process reducing the correlation between individual trees and improving the model's overall performance.

2.2 Room data set

In order to define a feasible strategy for deciding whether statistical formulae are applicable for a given room based merely on the room geometry and absorption properties as features, a reference room database is required serving as the basis of machine learning algorithms, for which both room parameters and reverberation parameters are known. However, room acoustic measurements and simulations both yield 'noisy' data sets in the context of accuracy. This can be accounted to the result of measurement noise, unknown or inaccurate material parameters, or simplifications in the modeling or simulation process, for example by assuming locally reacting materials described by the sound absorption coefficient instead of using extended reactivity, or geometrical simplifications, estimated scattering coefficients, numerical errors, partially implemented wave phenomena etc. Therefore, obtaining accurate reference data is practically not feasible. Despite this it is still possible to derive practically applicable results.

In our current approach the reference data was obtained using numerical calculations in Odeon Room Acoustics Software (version 17), in which a data set of almost 30,000 empty rectangular rooms of varying dimensions were created and evaluated.

The T_{20} reverberation time was taken as the 'ground truth' or reference reverberation time value in each room at each octave band.

For this study, we only considered empty rooms of typical sizes, but the rooms did contain objects such as doors, windows and covering materials on walls and ceilings of different dimensions. These elements had insignificant thickness compared to the wavelength. Both the dimensions of the rooms and the acoustic properties and coverings of the walls varied over a wide range from 1 to 100 meters and the materials were assigned sound absorption values of currently available, manufactured products. The data set contained both random size and distribution of covering materials as well as realistic scenarios such as a full or partial covering also considering currently available panel dimensions.

It is essential to acknowledge that the distribution of

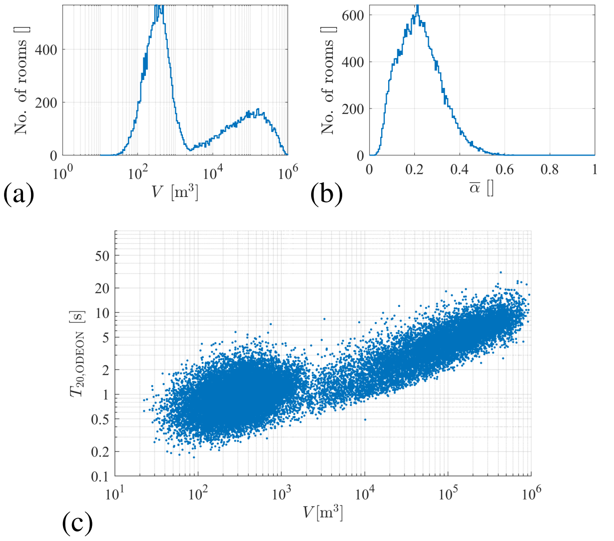

Figure 1. Histogram of the volume (a) and average absorption coefficient (b) in the input room dataset with the resulting input reverberation time versus the room volume (c)

both the room volume and the average room absorption parameter in the utilized room database is a recognized limitation but the method presented in this paper can be applied to other data sets as well. Based on the distribution of the current data set, it is expected that the reverberation time will be strongly correlated to the room volume, i.e., its geometrical properties, resulting in diminished machine learning algorithm performance. This property is also true for the statistical equations so the problem presented in this paper is considered 'difficult' for the machine learning approach.

Figure 1 provides an illustration of the distribution of the room volume, average absorption coefficient, and the corresponding T_{20}-vs-volume relationship, as depicted in panels (a), (b), and (c), respectively.

2.3 Target response function

We define the error function as the relative difference of the statistical and numerical results, as follows

$$E_{\mathrm{rel,stat}} = \sum_f \frac{|T_{20,\mathrm{reference}}(f) - T_{\mathrm{stat}}(f)|}{T_{20,\mathrm{reference}}(f)}. \quad (1)$$

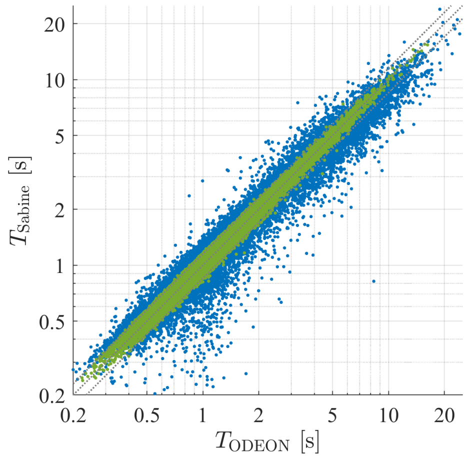

Figure 2. Scatter plot of the Sabine reverberation time versus the reference, numerical reverberation time for the room database. Green dots denote rooms for which the proposed validity requirements are met. Dotted lines denote the margin of errors, i.e. ± 15 %. Approximately 15,000 rooms are meeting the requirements (i.e. fulfilling $E_{\mathrm{rel,Sabine}} \leq 15\%$) and the same amount is outside of the error bounds.

In the present discussion the statistical result $T_{\mathrm{stat}}(f)$ may be obtained by applying any one of the statistical formulae (e.g., Sabine, Eyring, Arau, Fitzroy). For the reference results, an ensemble average of approximately 100 grid points (areas) were taken and the relative error was also averaged over the octave bands between 63 Hz and 8 kHz. The margin of error was defined as ± 15 % or a range of 30 %, that is, the statistical formulae was considered as applicable if the error defined as above was within this bound. This subjectively large bound was arbitrarily selected to slice the data set into equal number of true and false training sets and maximize number of data points, but the presented method of this paper allows for setting any other error bound, too.

The distribution of rooms, meeting the above requirement with the error definition applied for the Sabine formulation is depicted in Figure 2, containing around 15,000 rooms fulfilling $E_{\mathrm{rel,Sabine}} \leq 15\%$ and 15,000 room samples falling outside the bound.

2.4 Room features

In the present classification problem we aim at finding and ranking manually selected room features or properties for which a decision tree will deliver a proper decision at maximized probability. The inputs are manually selected features of the room, and outputs are the error of the assumption yielded by the statistical formula compared to a reference, and when within margin, the applicability is considered true.

Initially, 50 manually selected features of the individual input rooms were evaluated. The features (including that of the EN 12354-6 standard) were based on:

- **Geometrical properties of the room**: including the dimensions of the room, their various ratios, moments of their distribution up to the fourth order, and the mean free path; and
- **Distribution of the absorbing materials**: including moments of the absorption coefficients up to the fourth order, measures of inhomogenity of their spatial distribution (mean absorption of distribution in adjacent and neighbouring walls).

In the process of classification the number of possible input features was manually decreased from the initial 50 property vectors by removing less significant features based on feature ranking by ANOVA and Chi^2 analysis, and by manually filtering features based on partial dependencies.

3. RESULTS

3.1 Evaluation of statistical formulae

As a first glance to the problem we evaluated the reverberation time of our room data set based on different formulae including the EN 12354-6:2003 European standard [3] and its appendix, the Sabine formula, the Eyring formula, the Fitzroy equation, and the Arau equation. The resulting reverberation time are plotted in Figure 3 as the function of the reference reverberation times. The figure highlights that when numerical simulation results are considered as the reference, all the aforementioned formulations fall short in estimating the reverberation time of rooms with widely varying parameters. This also raises the question to what extent the room features in these equations can be considered valid or significant descriptors for ordinary spaces.

3.2 Performance evaluation of the current EN standard

According to Section 4.6 of the EN 12354-6:2003 European standard, the Sabine formula can be used to predict the reverberation time in empty rooms provided that the following specific conditions are all met.

- regularly shaped volumes: no dimensions should be more than 5 times any other dimensions
- evenly distributed absorption: absorption coefficient should not vary by more than a factor of 3 between pairs of opposite surfaces.
- the volume ratio is less than 0.2

This third factor is always true for empty (unfurnished) rooms, being 0. The Sabine formula is given by:

$$T_{60} = \frac{24 \cdot \ln 10 \cdot V}{c \cdot \sum_i \alpha_i \cdot S_i + 4mV} \tag{2}$$

Where the speed of sound is $c = 343$ m/s at 20 °C and V is the room volume. The absorption of air is defined in ISO 9613-1:1993 and is based on an empirical formula that is a function of relative humidity, temperature, atmospheric pressure, and frequency.

We evaluated the performance of the standard's three criteria by considering two aspects

- accuracy of choice of method: check how effectively the applied conditions split the room data set to Sabine-applicable and non-applicable groups
- overall accuracy: check if the non-applicable portion, when evaluated as in Appendix D, will deliver a result with less deviation than if they were evaluated with the statistical formulae only.

We first evaluated the probability of a proper decision based on the standard's conditions whether the Sabine formula is applicable or not within an approximately $\pm15\%$ of error margin compared to the reference data set. The resulting probability was 52 % which is very close to a random decision. On the other hand, when evaluating the splitting capabilities of the data set based on this decision, it also turned out that there were not only false positives but many false negatives, too, that is, cases where the Sabine formula was applicable but was deemed as non-applicable. This may drive an acoustical consultant's decision to use a more expensive calculation method even though it is not needed. This also suggests that both the the room features and their assigned limit values are likely

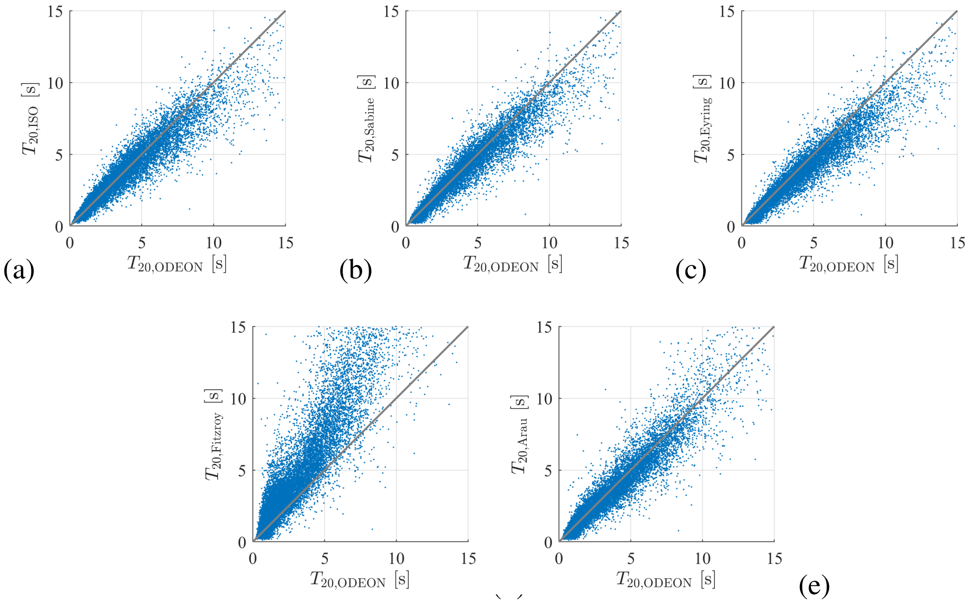

Figure 3. Result of commonly used statistical formulae for the reverberation time as the function of the reference, Odeon reverberation times. The applied statistical formulae are EN 12354-6:2003 European standard (a), Sabine (b), Eyring (c), Fitzroy (d) and Arau (e) formulations.

mismatched to an accurate decision, and they might also be too restrictive.

When the calculation of Appendix D of the standard is applied, it can be seen from Figure 3 (a) that the deviation has not decreased, so despite the complicated procedures there seems to be little merit in using this method in a consultant's task.

3.3 Improvement of the limitation criteria of the EN 12354-6 standard

3.3.1 Improvement using the standardized room features

A limited yet simple method to improve the performance of the decision in the standard is by keeping its room features but aligning the decision to hierarchy forming a decision a tree and setting up new values supporting a better decision. The current parallel true-false decision method based on features was can be significantly improved this way, even if the choice of room features are not optimal. The feature set this way includes the ratio of the longest and shortest dimensions (EN condition 1), the absorption coefficient ratio of opposite walls (EN condition 2 a-b-c), and the room volume ratio (EN condition 3).

Both bagged tree ensembles and simple, coarse decision trees were trained on 90 % of the total room dataset, with 10 % reserved for testing. The coarse decision tree structure produced a training accuracy of 59 %, which was surprisingly better than the more complex bagged tree ensemble, having a training accuracy of 57 %. The resulting coarse decision tree structure is depicted in Figure 5.

Note that the decision is based on EN condition 1 and EN condition 2b. For EN condition 1 the decision limit of 5 is recovered, which perfectly matches with the EN 12354-6 condition. Nevertheless, the simple decision tree aligns with the original EN 12354-6 conditions, which state that the Sabine formula is not applicable for rooms where the ratio of the longest and shortest dimensions is greater than 5.8. Yet, below this limit the ratio of dimensions does not contain enough information for a unequivocal decision.

The above statement is further verified by investigating the scattering diagram of the classification results. The room sample distribution for which the Sabine formula is applicable based on the decision of the coarse tree is depicted in Figure 6 (a-c). The decision tree can accurately detect outlier rooms for which the Sabine formula

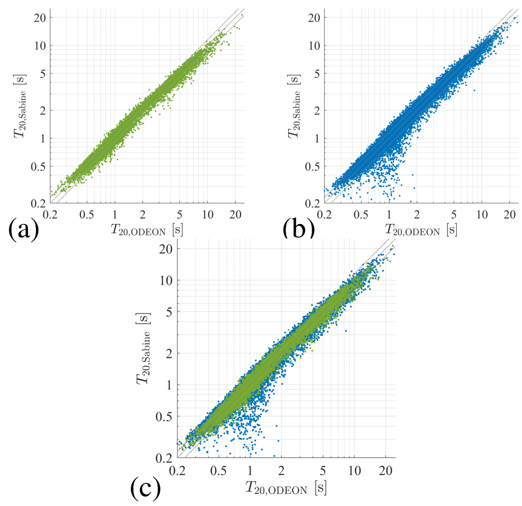

Figure 4. Scatter plot of the Sabine reverberation time versus the reference, numerical reverberation time for the room data set. Dotted lines denote the margin of errors, i.e. ± 15 %. Green dots denote rooms for which the EN 12354-6:2003 standard requirements are met to apply the Sabine formula. These criteria will predict the 'safe' applicability of the Sabine formula versus numerical modeling at no better than 52 % probability.

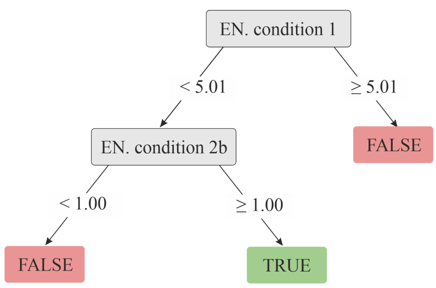

Figure 5. Coarse decision tree trained on the EN 12354-6 standard validity conditions

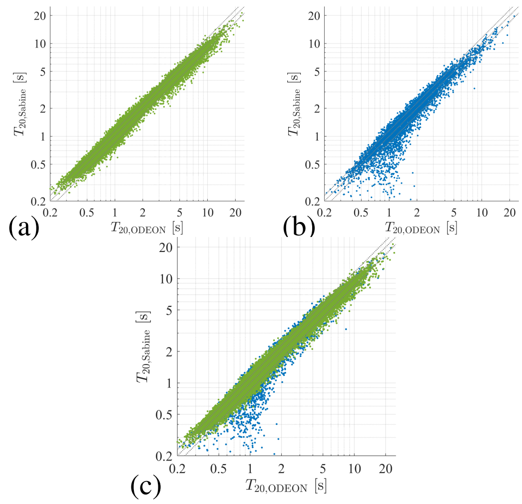

Figure 6. Result of the classification problem using a coarse decision tree, trained on the EN 12354-6 standard validity conditions. Again, dotted lines denote the margin of errors, i.e. ± 15 %.

is not valid. The improvement of accuracy is due to the more precise detection of false-positive errors in comparison with 4. Despite the improved probability of decision, this method is still prone to false-negative errors, similar to the original parallel decision.

3.3.2 Improvement using new features

Since the improvement of the decision based on the standardized room features seem to be rather limited, we listed 50 geometrical and absorption-type features and conducted an ANOVA analysis to find potentially more relevant features. As the outcome of analysis three features allowed achieving a training accuracy of 70 % by using a simple coarse decision tree. Neither including more absorption-type features, nor the application of more complex tree structures increased the accuracy of the training. The new, proposed features are

- The standard deviation of the room dimensions
- The mean free path, defined as [4]

$$\overline{d} = \frac{4V}{S}, \tag{3}$$

where V is the total volume of the room, and S is total surface area enclosing the room.

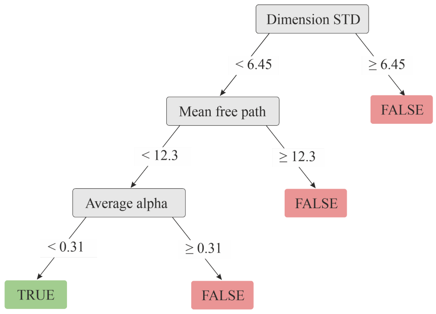

Figure 7. Coarse decision tree trained on the new, proposed feature set.

- The average absorbing coefficient of the room walls
$$\overline{\alpha} = \frac{A}{S}, \tag{4}$$
with A being the equivalent absorbing area of the total room surface.

The simple coarse decision tree is depicted in Figure 7. It is further verified that the basis of decision mainly relies on the standard deviation of the room dimensions, and this statistical moment yields a more reliable upper limit for the usability of the Sabine formula than dimension ratios.

Again, as a further aspect the distribution of the classified rooms are investigated on a scatter plot in Figure 8. Similarly to the previous case by using the EN standard-based feature set, the increase of classification accuracy is due to improved false-positive error detection. It can be seen that rooms with reverberation time less than 3 seconds can be separated more precisely into Sabine-applicable and non-applicable classes compared to previous methods and the current feature set of the EN 12354-6 standard. We verified the usability of the proposed decision tree up to about 3 seconds of reverberation time – from this value up, the decision will always be to not-applicable (for using the Sabine formula). Although this is in line with current room acoustic consulting practice, a wider range of training data will likely extended the validity range of a decision tree based on that data. In the current data set there are few highly overdamped or underdamped room samples, as shown in Section 2.2. This is also verified by examining Figure 1 (c), highlighting that the training dataset contains no small and medium-large sized rooms with reverberation time above 3 seconds. We

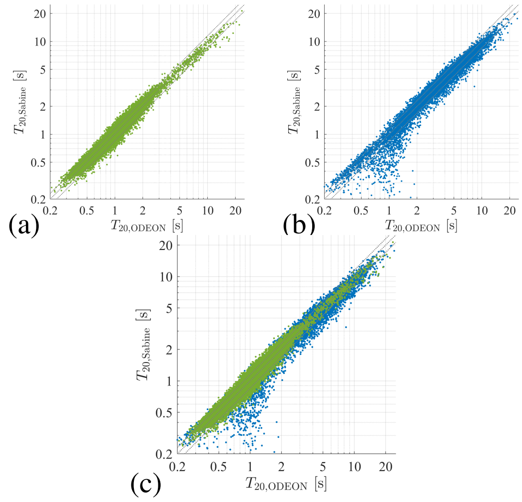

Figure 8. Result of the Sabine classification problem using a coarse decision tree, trained on the proposed geometrical feature set.

believe that introducing more training data and room features this results can be further improved, being the subject of upcoming research.

Finally, Table 1 summarizes the results by comparing the training and testing accuracy of the original standard requirements along with the proposed, decision tree based strategies. It is highlighted that even a simple coarse decision tree with the feature set being the original EN 12354-6 standard requirements (shown in Figure 5) may increase the decision accuracy by about 7 % compared to the currently standardised limitations. Still, it was shown that this decision relies merely on the ratio of the room dimensions. By applying the proposed geometrical feature set, composed of the standard deviation of the room dimensions and the mean free path, a better decision tree could be given — depicted by Figure 7 —, increasing the decision accuracy to about 72 %.

Table 1. Prediction accuracy of the applicability of the Sabine formula

	Training/Testing
Current EN standard	52% / -
Proposed tree (EN features)	59%/ 61%
Proposed tree (new features)	70% / 72%

3.3.3 *Applicability to further statistical formulae*

The present approach can be applied and extended to other statistical reverberation formulae such as the Norris-Eyring (or Eyring), Arau-Puchades [5] (or Arau) and Fitzroy [6] formulae, and many others. This subsection describes the application of the feature sets discussed earlier to classify the suitability of the Eyring, Arau, and Fitzroy formulations for a particular room geometry. To accomplish this, the relative error (1) was computed between the reference reverberation time and that of the statistical formula. As before, the classification threshold was set at an error margin of 15 %.

Table 2. Prediction accuracy of the applicability of various statistical reverberation formulae using the presently proposed decision tree method

	EN features	Proposed features
Sabine	61%	72%
Eyring	59%	72%
Arau	58%	62%
Fitzroy	56%	83%

Table 2 summarizes the results of the coarse tree classification of the other statistical formulae. It is notable that the applicability of the Sabine and Eyring formulae for this data set showed a very similar prediction accuracy and a higher performance was achieved for the Fitzroy equation suggesting its more limited applicability. Apparently these room features cannot reliably predict the applicability of the Arau formula.

4. CONCLUSIONS AND FUTURE WORK

In conclusion, this study proposed a new method for determining the applicability of statistical reverberation formulae to a given room geometry using machine learning tools.

The classification was based on the relative error between the reference reverberation time obtained from numerical simulations using the Odeon Room Acoustics Software and the reverberation time predicted by various statistical formulae. The accuracy of the classification was improved by training coarse decision trees on a database of nearly 30,000 rooms. In this study, a relative prediction error below 15 % was considered as the boundary of applicability.

For the Sabine equation, three simple features were identified, which allowed more accurate classification of rooms with a reverberation time below 3 seconds, significantly surpassing the accuracy of the presently standardised classification scheme. The 3-second threshold was attributed to the composition of the training data set.

It was also shown that the presented new feature set allows the investigation of applicability of other frequently used statistical formulae, e.g. the Eyring or Fitzroy formulae.

The presented method is implemented in the soundy.ai application by providing suggestions to the acoustical consultant highlighting whether the predicted result using a given statistical reverberation time formula is likely reliable or not.

In our future work we plan to extend the room database and apply the presently proposed method to refine the presently proposed decision tree, by considering further rectangular rooms and furniture, as well as other room shapes.

5. ACKNOWLEDGMENTS

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6. REFERENCES

[1] H. Kuttruff, *Room Acoustics*. CRC Press, 2017.

[2] C. Eyring, “Reverberation time in ”dead” rooms,” *Journal of the Acoustic Society*, 1930.

[3] “EN 12354-6:2003,” standard, European standards, 2003.

[4] C. W. Kosten, “The mean free path in room acoustics,” *Acoustica*, vol. 10, pp. 245–250, 1960.

[5] H. Arau-Puchades, “An improved reverberation formula,” *Acoustica*, vol. 65, pp. 163–180, 1988.

[6] D. Fitzroy, “A reverberation formula which seems to be more accurate with a non-uniformdistribution of absorption,” *J. Acoust. Soc. Amer.*, vol. 31, p. 893, 1959.