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NOVEL VIBROACOUSTIC METHOD FOR SENSORLESS MEASUREMENT OF COGGING TORQUE OF PERMANENT MAGNET SYNCHRONOUS MOTORS

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In this paper a new vibroacoustic modelling methodology of permanent magnet synchronous motors (PMSMs) is presented. The model accurately describes torsional vibration conditions of a PMSM in case of single-phase pulsating current excitation, in conjunction with small angular displacement of the rotor. The step-by-step derivation of the model, the method of model parameter identification based on electrical impedance measurements and the calculation of model parameters are investigated in details. Special attention is paid to the vibroacoustic effect of cogging torque, and using the theoretical results the possibility of sensorless cogging torque measurement is mentioned.

1. Introduction

In acoustics there are a number of examples for such systems connecting different fields of physics. An excellent example is the direct radiator dynamic loudspeaker, in which three different fields of physics are coupled in one device. From one hand the electric side (e.g. the voltage and current of the voice coil) is coupled to the mechanical side (the velocity of the diaphragm and the force acting on the diaphragm) by means of the interaction between the air-gap flux and the voice coil current, and on the other hand the mechanical side of the speaker is coupled to the acoustic field (sound pressure and particle velocity) by means of sound radiation of the vibrating diaphragm.

In enclosure design the use of the lumped equivalent network of the loudspeaker and the coupled acoustic system is a very efficient approach, therefore we are trying to adopt this methodology to torsional systems, namely to permanent magnet synchronous motors (PMSMs). The similarity between these two different devices becomes obvious when the equivalent networks are compared to each other, however in some cases the physical meaning of the model parameters is different.

In the first part of the article the well known equations of a PMSM are recalled, and on the basis of these equations and some illustrative considerations the vibroacoustic model is derived step-by-step. Using this vibroacoustic model the electrical impedance of a PMSM is calculated and a parameter identification method is presented that can estimate measured impedance curves by remarkable accuracy. Finally the possibility of physical parameter calculation from transfer function coefficients is investigated and the equations for the parameters are given. In the end, the calculated parameters are presented for a medium power (~1 kW) PMSM and the possibility of sensorless cogging torque measurement is mentioned.

2. The standard model of a PMSM

In this section the standard model of a PMSM (Fig. 1.) is briefly summarized.

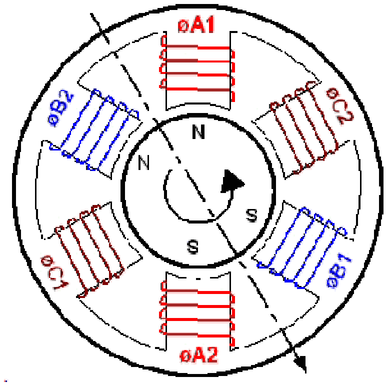

Figure 1. A schematic section view of a 2 pole 3 phase PMSM[1].

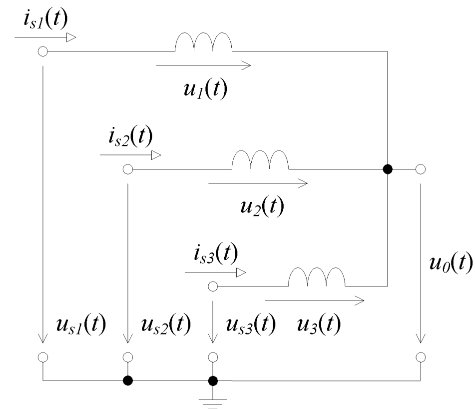

Figure. 2. The schematic circuit diagram of a PMSM with wye-connected phases.

The electrical behaviour of permanent magnet synchronous motor can be summarized in the following equations[2]:

$$u_{si} - u_0 = R_s i_{si} + L_s \frac{di_{si}}{dt} + u_i , \tag{1}$$

$$\sum i_{si} = 0 , \tag{2}$$

where i = 1, 2, 3, and u_{si}, i_{si} are the stator voltage and current of the i^{th} phase, u_i, u_0 are the induced voltage of the phases and the voltage of the common end of the windings (see Fig. 2.), and R_s, L_s are the resistance and inductance of the phases respectively. This model presumes that the rotor-position dependence of the inductances and the coupling between windings is negligible. In most cases – for instance in the design of a closed loop control – this negligence is permissible.

The induced voltage can be expressed from the flux change:

$$u_i = \frac{d\psi_{ir}}{dt} = \frac{\partial \psi_{ir}}{\partial \varphi}\frac{d\varphi}{dt} = \theta \frac{\partial \psi_{ir}}{\partial \varphi} , \tag{3}$$

where ψ_{ir} is the rotor flux in the i^{th} winding, φ is the angular displacement, and θ is the angular velocity. The torque acting on the rotor can be calculated from the phase currents and from the flux:

$$m = \sum_{i=1}^{3} m_i = \sum_{i=1}^{3} i_{si} \frac{\partial \psi_{ir}}{\partial \varphi} . \tag{4}$$

The structural dynamics of the motor is described by the following two equations:

$$\frac{d\varphi}{dt} = \theta , \ \frac{d\theta}{dt} = \frac{1}{J_r}\left(m - m_l - m_c - B\theta - C sign(\theta)\right), \tag{5}$$

where J_r is the moment of inertia of the rotor, m_l is the load torque, m_c is the cogging torque, B is the viscous friction coefficient, and C is the Coulomb friction coefficient.

The cogging torque is due to the attraction of the permanent magnets and to the salient pole pieces of the stator iron, therefore the cogging torque is always present, even in the absence of the phase currents.

It is common in practice to express the non-sinusoidal quantities ψ_{ir} and m_c by their Fourier series, but the following calculations do not use this approach.

3. The vibroacoustic model of a PMSM

In this section the step-by-step derivation of the vibroacoustic model from the standard model is presented. In our case the "vibroacoustic modelling" means that the rotor of the PMSM does not whirl but rather oscillates torsionally around an equilibrium position with considerably small amplitude.

During the modelling it is assumed that the rotor, the shaft and the housing is ideally rigid, however it is mentioned that the methodology presented here can be applied to the vibroacoustic investigation of the components listed above.

3.1 Modelling of coupling

According to the example mentioned in the introduction in Fig. 3. the electromechanical transducer of a loudspeaker is shown.

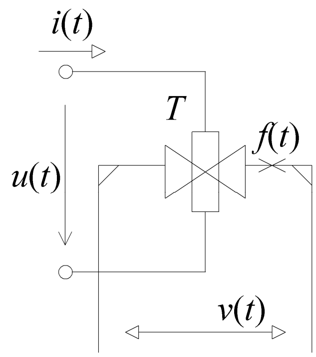

Figure 3. The magnetic circuit of a dynamic loudspeaker as an electromagnetic transducer.

The well known governing equations of the transducer are the following[3]:

$$u(t) = Blv(t) = Tv(t)\,, \tag{6}$$

$$f(t) = Bli(t) = Ti(t)\,, \tag{7}$$

where $u(t)$ and $i(t)$ are the voltage and current of the voice coil, and $f(t)$ and $v(t)$ are the force acting on the voice coil and the velocity of the voice coil respectively. The magnetic circuit is usually characterized by the Bl product (where l is the length of the wire wound on the voice coil and B is the induction in the air gap) that is often referred as T electromagnetic transmission.

According to the everyday acoustic practice it would be convenient to use the same transducer model in case of a PMSM too. Eqs. (3) and (4) inherently give the opportunity to do so, the only thing that has to be done is to apply the assumption of small angular displacements. Under such conditions the derivatives of flux can be rewritten as angular position dependent constants:

$$u_i = \theta \frac{\partial \psi_{ir}}{\partial \varphi} = \theta T(\varphi)\,, \tag{8}$$

$$m = i_{si} \frac{\partial \psi_{ir}}{\partial \varphi} = i_{si} T(\varphi)\,. \tag{9}$$

These equations describe the relationship between the electrical and the mechanical side of the motor, and compared to the loudspeaker the only difference is that instead of force and linear velocity those contain torque and angular velocity.

3.2 Modelling the electrical components

According to Eq. (1) the modelling of the electrical components is rather obvious, the equivalent network should be amended with a resistor and an inductor. In vibroacoustic modelling the position dependence of the inductance is not negligible therefore we introduce $L(\varphi)$ position dependent inductance. The equivalent circuit with these two elements and with the electromagnetic transducer can be seen in Fig. 4.

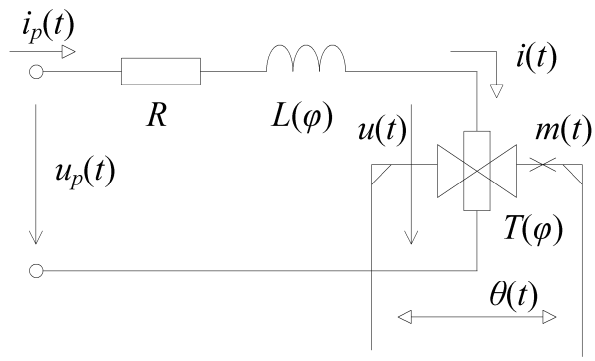

Figure 4. The vibroacoustic model of a PMSM containing the electronic components and the electromagnetic transducer.

3.3 Modelling the mechanical components

The further investigations are continued in the complex frequency domain and the Laplace transform of quantities is denoted with capital letters and *s* arguments.

Derivation of the mechanical components is also rather straightforward, however to keep the formalism conform with the one used in standard acoustic calculations, torsional mechanical impedance is introduced according to the following definition:

$$Z_t(s) = \frac{M(s)}{\Theta(s)}. \tag{10}$$

Using this definition and Newton's second law the torsional mechanical impedance of the moment of inertia of the rotor can be written as:

$$M(s) = J_r \beta(s) = J_r s\,\Theta(s) \;\rightarrow\; Z_{tJ_r}(s) = \frac{M(s)}{\Theta(s)} = sJ_r. \tag{11}$$

The vibroacoustic effect of the cogging torque can be derived according to the following concept. Let us write the cogging torque as an arbitrary function of the rotor position:

$$m_c = m_{cg}(\varphi)\ \left[Nm\right], \tag{12}$$

and write the derivative of it with respect to the rotor position:

$$\frac{dm_c}{d\varphi} = \frac{dm_{cg}(\varphi)}{d\varphi}\ \left[\frac{Nm}{rad}\right]. \tag{13}$$

Eq. 13 shows that the derivative of cogging torque is a 'torsional stiffness like' quantity, because its unit is [*Nm/rad*], therefore the effect of cogging torque can be taken into account as a position dependent torsional spring with a

$$K_{tc}(\varphi) = \frac{dm_{cg}(\varphi)}{d\varphi}\ \left[\frac{Nm}{rad}\right] \tag{14}$$

stiffness.

In the field of electroacoustics instead of stiffness, compliance is very often used to measure the rate of a spring, because in that way electrical equivalent networks can be drawn more easily.

Considering Eq. (13) one can write the mechanical impedance of the equivalent torsion spring of cogging torque:

$$Z_{tc}(j\omega,\varphi)=\frac{K_{tc}(\varphi)}{s}=\frac{1}{sC_{tc}(\varphi)}. \tag{15}$$

At this point at the mechanical side there is a torsional mass and a spring coming from the inertia of the rotor and from the cogging torque respectively. Mechanical losses such as the Coulomb and the viscous friction, as well as the magnetic losses will probably damp this single degree of freedom mass-spring system, hence damping can be taken into account most easily by a torsional resistor:

$$Z_{tr}(s)=R_t. \tag{16}$$

3.4 The complete vibroacoustic model of a PMSM

Fig. 5. shows the complete vibroacoustic linear single-phase lumped equivalent network of a permanent magnet synchronous motor, consisting of a series resistance and inductance for modelling the winding, an electromagnetic transducer for modelling the coupling, the moment of inertia of the rotor, a torsional spring for modelling the cogging torque and a torsional resistor for modelling the mechanical and magnetic losses.

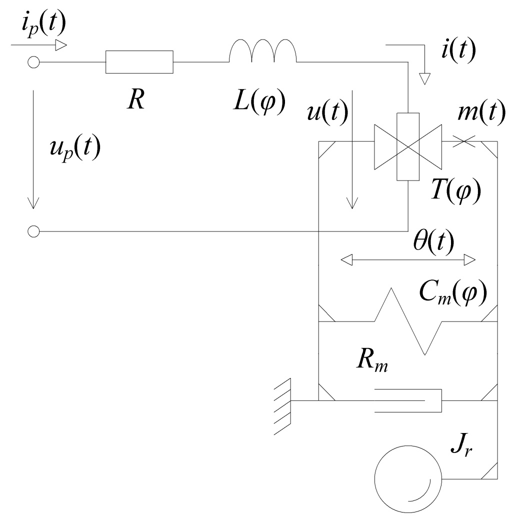

Figure 5. The complete vibroacoustic linear single-phase lumped equivalent network of a permanent magnet synchronous motor.

4. Parameter identification

4.1 Electrical impedance calculation

To calculate the electrical impedance of the motor let us first examine, how the electromechanical transducer transforms electrical and mechanical impedances. Inserting Laplace transforms of Eqs. (8) and (9) to the expression of electrical impedance, and using the definition of torsional mechanical impedance (Eq. (10)) one can get the following equation:

$$Z_{em}(s)=\frac{U(s)}{I(s)}=\frac{\Theta(s)T(\varphi)}{\frac{M(s)}{T(\varphi)}}=T^2(\varphi)\frac{\Theta(s)}{M(s)}=\frac{T^2(\varphi)}{Z_m(s)}. \tag{17}$$

As one can see, the motor transforms the impedance from one side to the other reciprocally, multiplied by the square of the transmission ratio.

As far as the mechanical components have the same angular velocity, the mechanical impedance can be written as:

$$Z_m(s) = \frac{1}{sC_m(\varphi)} + R_m + sJ_r = \frac{1 + sR_m C_m(\varphi) + s^2 J_r C_m(\varphi)}{sC_m(\varphi)}. \tag{18}$$

Using the expression of mechanical impedance and the relationship among impedances, one can write the electrical impedance of the motor:

$$Z_e(s) = R + sL + Z_{em}(s) = \frac{R + s(C_m R_m R + L + T^2 C_m) + s^2 C_m (J_r R + R_m L) + s^3 J_r C_m L}{1 + sC_m R_m + s^2 J_r C_m}. \tag{19}$$

4.2 Curve fitting

Transfer function models were fitted on measured impedance data by using MATLAB S*ystem Identification Toolbox* (Ident), and the parameters of the Output Error (OE) model were estimated in the frequency domain[4]. In Ident parameters of unstable systems cannot be estimated, hence the identification was performed on the electrical admittance rather than the impedance.

In Ident the leading coefficient of the denominator of a transfer function model is equal to 1, therefore one can express the impedance as a transfer function in the following form:

$$\begin{aligned} Z_e(s) &= \frac{R + s(C_m R_m R + L + T^2 C_m) + s^2 C_m (J_r R + R_m L) + s^3 J_r C_m L}{1 + sC_m R_m + s^2 J_r C_m} = \\ &= \frac{A(s)}{B(s)} = \frac{a_1 + a_2 s + a_3 s^2 + s^3}{b_1 + b_2 s + b_3 s^2} = \frac{A_1 + A_2 s + A_3 s^2 + A_4 s^3}{1 + B_2 s + B_3 s^2}. \end{aligned} \tag{20}$$

Fig. 6. shows measured impedance curves of a medium power (~1 kW) 10 pole 12 slot PMSM. Beside the well known effect of the inductance (ie. the impedance is increasing by 20 dB/decade beyond the corner frequency of the R-L circuit) the peak of the rotor – cogging torque resonant system is quite significant. The change of coupling between the electrical and the mechanical side is also noticeable, moreover at 13.2° it becomes zero. The correspondence of measured and estimated impedance curves is very convincing, showing that the order of the theoretical model is appropriate.

4.3 Parameter calculation

From the estimated transfer function coefficients the model parameters can be calculated solving a set of equations, however these equations are linearly constrained and the number of unconstrained equations is fewer by one than the unknowns to be determined. This means that all the unknowns of the system can be calculated only if one is given. Here it is most efficient if one chooses the moment of inertia of the rotor as the given value, since it certainly does not depend on φ. Therefore the following equations yield the relationships between the coefficients of the frequency response function and model parameters:

$$R = A_1, \; L(\varphi) = \frac{A_4}{B_3}, \; T(\varphi) = \sqrt{\frac{J_r}{B_3}\left(A_2 - A_1 B_2 - \frac{A_4}{B_3}\right)}, \tag{21} (22) (23)$$

$$C_m(\varphi) = \frac{B_3}{J_r}, \; R_m = \frac{B_2 J_r}{B_3}. \tag{24} (25)$$

The calculated electrical and mechanical parameters of a medium power (~1 kW) PMSM are shown in Fig. 7. and Fig. 8. respectively. Due to low frequency estimation error in the transfer functions the electrical resistance has some position dependence, that can be avoided by global optimization of transfer function coefficients and the use of a single R value. The inductance variation is closely sinusoidal with a mean value of around 180 μH that matches the measured inductance well.

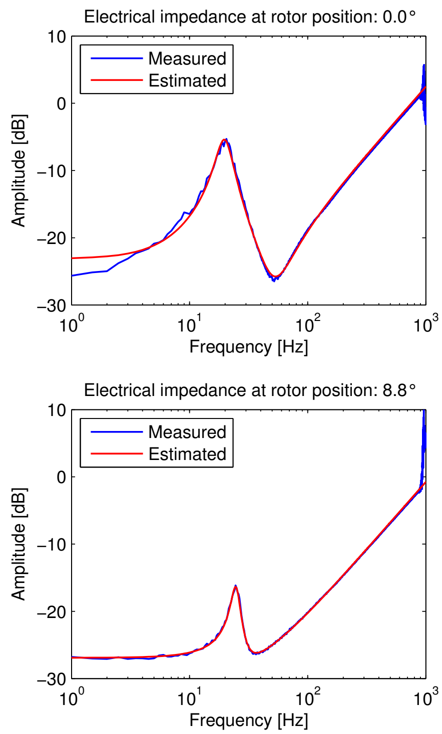

Figure 6. Measured electrical impedance curves in four different rotor position.

The absolute sine function shape of the electromagnetic transmission is also corresponding well with the expectations while the induced voltage of the motor is closely sinusoidal.

The shape of the torsional compliance and torsional resistance curves has to be analysed further, but the effect of the 10 pole of the motor is significant. The fact that the torsional loss varies according to the number of poles and the peak variation equals to the minimum value indicates that probably the magnetic and mechanical losses have the same order of magnitude, hence none of them can be neglected.

The inversion of the relationship between cogging torque and torsional compliance (Eq. (12)-(15)) provides the possibility of sensorless measurement of cogging torque based on vibroacoustic principles.

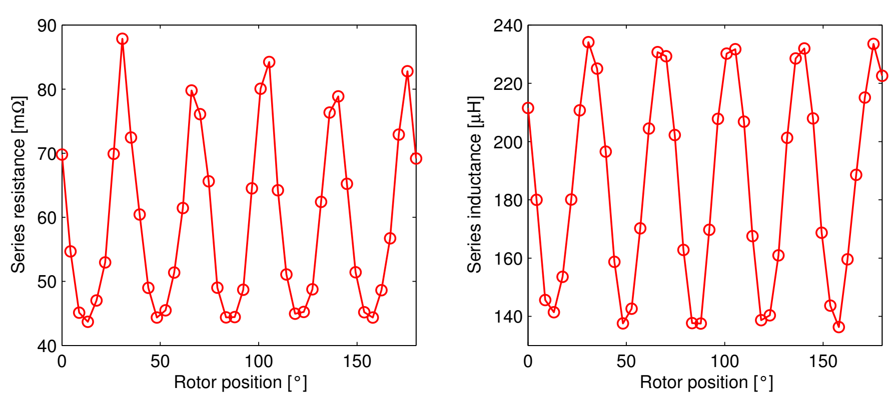

Figure 7. Calculated electrical parameters as a function of rotor position.

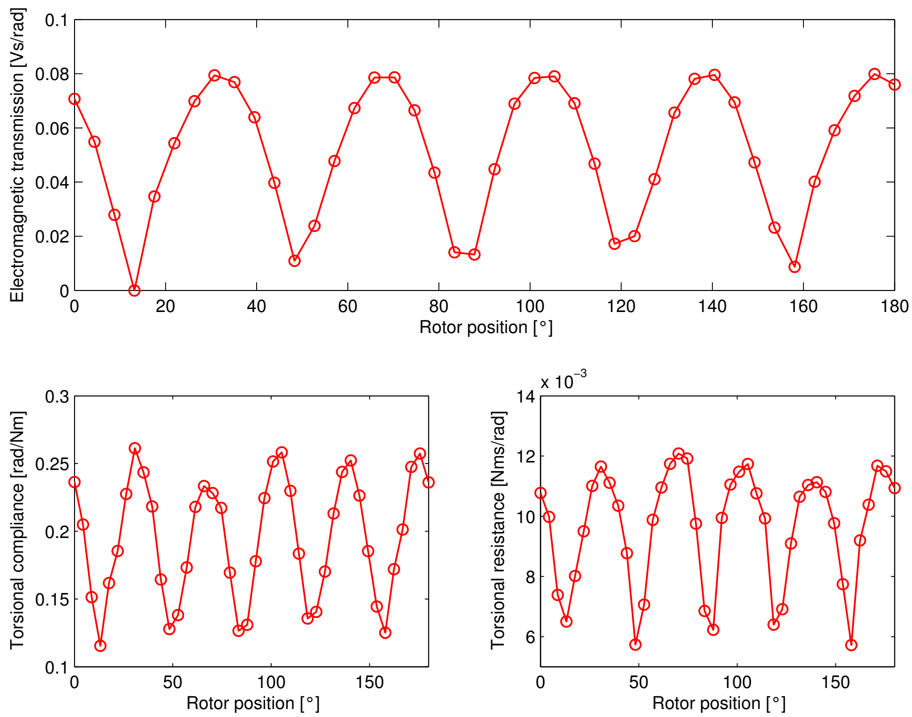

Figure 8. Calculated mechanical parameters as a function of the rotor position.

5. Summary and future work

In this paper a novel vibroacoustic model for permanent magnet synchronous motor is outlined. In the first part the standard model of the PMSMs is introduced and using these equations the vibroacoustic model can be derived. This model can predict the shape of the electrical impedance curve of the motor, and it was shown that from estimated transfer function coefficients the model parameters can be calculated.

The next step is to check the theoretical relationship between cogging torque and torsional compliance, and to investigate the variation of torsional loss in detail.

6. Acknowledgements

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REFERENCES

1. Source of Fig. 1.: http://www.basilnetworks.com/article/motors/brushlessmotors.html (downloaded on 20.04.2009.)
2. Kapun A., Curkovic M., Hace A., Jezernik K., Identifying dynamic model parameters of a BLDC motor, *Simulation Modelling Practice and Theory*, Article in Press, 2008.
3. L. L. Beranek, *Acoustics*, McGraw-Hill Book Company, Inc., New York / Toronto / London (1954).
4. L. Ljung, *System identification: Theory for the User,* Prentice Hall Ptr., Upper Saddle River, NJ, USA (1999).