Calculation of re-radiated noise in buildings, generated by underground rail traffic

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Abstract

The vibration generated by railway traffic causes complaints not only about sensible vibration levels, but about significant noise levels in the building lying beside or above rail lines. Therefore numerical methods to predict the effect of train passbys are very important to possess and to develop.

This paper presents a new numerical prediction method based on the Rayleigh radiation integral which allows us to calculate sound pressure levels inside a cavity when the normal velocity distribution along the boundary surfaces are known.

For validation the method was compared to the Boundary Element Method by means of calculations on theoretical rooms with theoretical velocity distributions along their walls.

Beside these calculations, thorough measurements were carried out in buildings - mostly monolythic concrete portal frame structure buildings. Synchronous, multi channel recordings of vibration and noise signals were performed. The results of the measurements and the simulations showed good agreement and are also presented in this paper. Both the development and the validation of the new method was done within the CONVURT project.

1 Introduction

To predict interior noise levels generated by railway traffic is a complex task. Beyond the proper understanding and characterisation of the source of vibration, the entire path of vibration propagation through different media and the mechanisms of sound radiation have to be modelled. In this paper the last stage of the prediction process, the calculation of re-radiated noise is presented.

The frequency range of the vibration generated by railway traffic and that of the re-radiated noise is typically limited to the lower frequencies - the dominant part is below 200 Hz. The primary input data for the calculation of sound radiation is the distribution of vibration velocities along the boundary surfaces of the cavity. This input is typically the output of a finite element based prediction. The most accurate and most common numerical way of predicting the radiated noise is using the boundary element method (BEM) [1, 2, 3]. The disadvantage of it is that BEM is a low-frequency method. Already in the case of a typical room, the size of the cavities, in which the radiated noise has to be computed, are generally too large for a computation efficient application: for calculations in the mid-frequency range, the resolution of the boundary mesh should be increased and that would lead to the problem of solving a large system of linear equations. The inversion of large matrices is very time consuming and despite the rapid development of the size of the memory modules swapping slows down the process even more.

Radiated noise can also be calculated with analytical formulas. The pivotal point of this method is the need of radiation efficiency. Radiation efficiency varies rapidly in the low-frequency range and is hard to determine precisely. Compared to BEM, these formulas are very simple, and give only rough approximations of noise levels.

The aim of the research was to develop a simple and robust method for predicting radiated noise field directly from the vibration data, without the need of constructing and inverting large matrices.

Rayleigh integral based calculations are well known and widely used to solve exterior sound radiation problems. This paper introduces a method that is also based on the Rayleigh integral, but uses a modified Green's function in order to predict interior noise levels from wall vibrations.

2 The description of the Rayleigh integral based method

If we know the surface normal component of the vibration velocity distribution along an infinite vibrating plane ($v_n(\boldsymbol{r})$), we can use the Rayleigh integral to calculate the radiated sound pressure in an arbitrary point ($\boldsymbol{q}$) of the space [4, 5]:

$$p(\boldsymbol{q},\omega) = \iint_\Sigma g(\boldsymbol{r},\boldsymbol{q},\omega) v_n(\boldsymbol{r}) d\Sigma \tag{1}$$

where

$$g(\boldsymbol{r},\boldsymbol{q},\omega) = \frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r})}}{|\boldsymbol{q}-\boldsymbol{r}|} \tag{2}$$

is the Green's function of the homogeneous acoustical full-space, $k = \omega/c$ the wave number, ω is the circular frequency, and c is the speed of sound.

If we apply the Rayleigh integral to the case of closed spaces, we get a rough estimation of the radiated sound pressure. The result is the sum of sound pressures radiated by each wall to an acoustical half-space. Each wall is handled separately as part of an infinite plane. Only a small part (ie. the wall) of the infinite plane vibrates, the surface normal vibration velocity along this part is known, and $v_n = 0$ otherwise:

$$p(\boldsymbol{q},\omega) = \sum_{n=1}^{N} \iint_{\Sigma_n} g(\boldsymbol{r},\boldsymbol{q}) v_n(\boldsymbol{r}) d\Sigma \tag{3}$$

where N is the number of walls.

To take into account not only the primarily radiated sound but the reflections from the walls, we have to modify the Green's function (2). Each reflection can be handled as a direct sound arriving from a new sound source (an image source), the location of which is determined by mirroring the originally radiating part of wall to the reflecting walls, and the amplitude of which is reduced proportionally to the absorption at the reflections. If we sum all the reflected sounds having a not neglectable level and the direct sounds, we get the radiated sound pressure level in the enclosure.

The direct implementation of this theory would result a complicated method, for a large number of image sources has to be taken into account and also their location and visibility has to be determined individually.

For simplicity, our investigations are restricted to the case of shoebox shaped rooms. This restriction enables us to neglect visibility tests and the determination of the location of the image sources can be easily automatized.

With the assumption that the absorption of the walls is constant, the Green's function can be further modified (2) and it can be written as:

$$\hat{g}(\boldsymbol{r}_0,\boldsymbol{q},\omega) = \frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_0)}}{|\boldsymbol{q}-\boldsymbol{r}_0|} + \Gamma(\omega)\sum_{k=1}^{K_1}\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_k)}}{|\boldsymbol{q}-\boldsymbol{r}_k|} + \cdots + \Gamma(\omega)^m \sum_{k=1}^{K_m}\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_k)}}{|\boldsymbol{q}-\boldsymbol{r}_k|} + \dots \tag{4}$$

where $\boldsymbol{r}_0$ is the location of the primary radiator, $\boldsymbol{r}_i$ is the location of the i-th image source, $\Gamma(\omega)$ represents the constant absorption and K_i is the number of the i-th order reflections.

The surface integrals in equation (3) are performed numerically. The whole surface is divided into surface elements of constant velocity distribution. The integration of the Green's function over the elements can be carried out by means of Gaussian Quadrature integration. The discretized form of (3) is:

$$p(\boldsymbol{q},\omega) = \sum_{i=1}^{N_e} v_{n_i} \sum_{s=1}^{N_p} w_s \hat{g}(\boldsymbol{r}_{i,s}, \boldsymbol{q}, \omega) \tag{5}$$

where N_e is the number of elements, N_p is the number of Gaussian points, v_{n_i} is the surface normal velocity of the i-th element, w_s is the Gaussian weight and $\boldsymbol{r}_{i,s}$ is the location of the s-eth Gaussian point on the i-th element.

3 Results of numerical testing

To verify and validate the newly designed prediction method, numerical tests were performed by means of shoebox shaped room of size $4\ \mathrm{m} \times 5\ \mathrm{m} \times 3\ \mathrm{m}$ in the x, y and z directions. The method of the numerical testing was to calculate the internal noise in different points by means of a direct BEM and the Rayleigh-based method, assuming several different normal velocity distributions on the walls. The calculations have been performed in a frequency range: $f \in [10\ \mathrm{Hz}, 120\ \mathrm{Hz}]$, using $N_e \approx 1500$ surface elements for both methods [6]. Figure 1 shows the eigenfrequencies of the test room within the frequency range of the calculations.

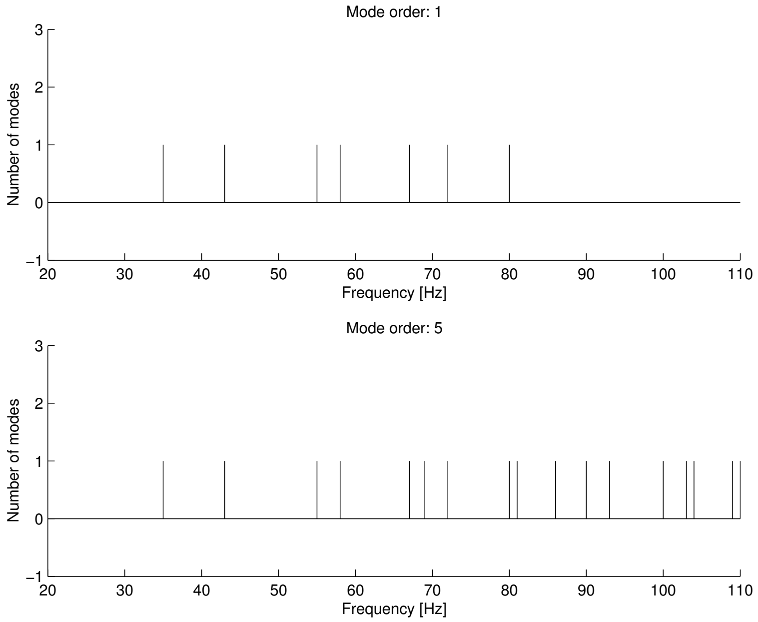
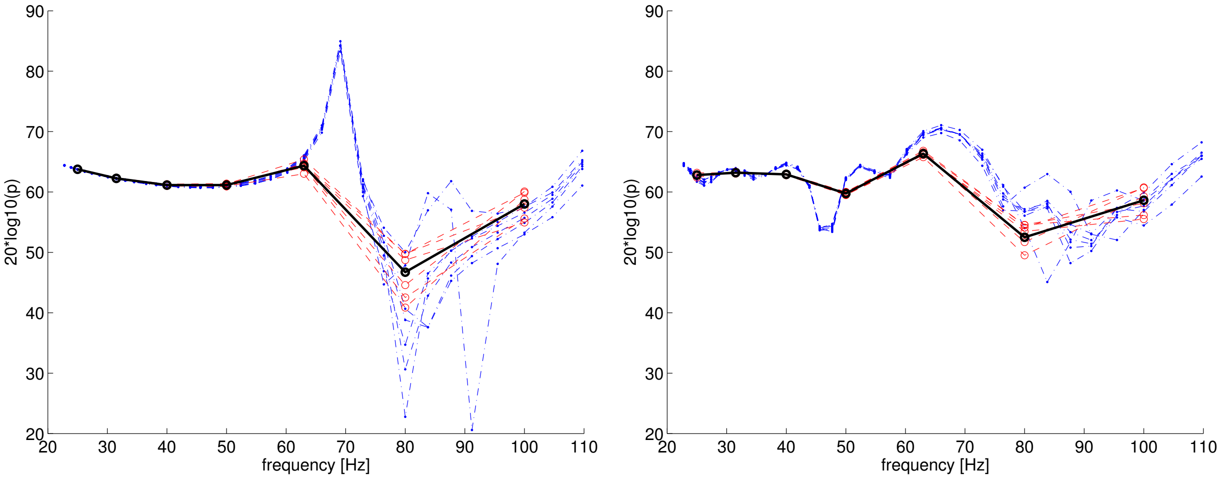


Figure 1: Distribution of eigenmodes of the test room in the frequency range of the calculations. The upper figure shows the first order modes, and the lower figure shows the modes up to fifth order

In the first tests a simple case was examined to investigate the convergence of the new method. One surface element was chosen as the only vibrating wall section of the testroom and the sound field generated by this single element was calculated for different absorption coefficients and with different number of reflections taken into account. Absorption was changed from 0 to 50 percent and number of reflections were increased from 0 up to 9261 (zero reflections means applying the original Rayleigh integral to the walls). The results showed what had been expected: with increasing absorption the fluctuations in the response decrease, and with increasing the number of reflections the modal behaviour of the room can be observed more and more clearly.



(a) BEM (b) Rayleigh

Figure 2: The effect of spectral and spatial averaging on the noise spectrum for the case of the BEM and the Rayleigh-based calculations

In the second series of tests the effects of spectral and spatial averaging (dithering) were examined. Figure 2 shows narrow and third octave band spectra of noise levels calculated in seven points located closely around one internal point, assuming constant velocity distribution all over the room's surface. The maximum distance between two points is 0.2 m. The blue dashed lines show the narrow band spectra, the red dashed curves show the third-octave band spectra and the black solid line shows the third octave band spectrum averaged to the seven points. Although the narrow band spectra can show significantly different levels, the spectral averaged curves are in good agreement. The necessity of spatial averaging is also shown. The presence of the room eigenfrequencies can be observed at 70 Hz, 80 Hz, 90 Hz (see also figure 1).

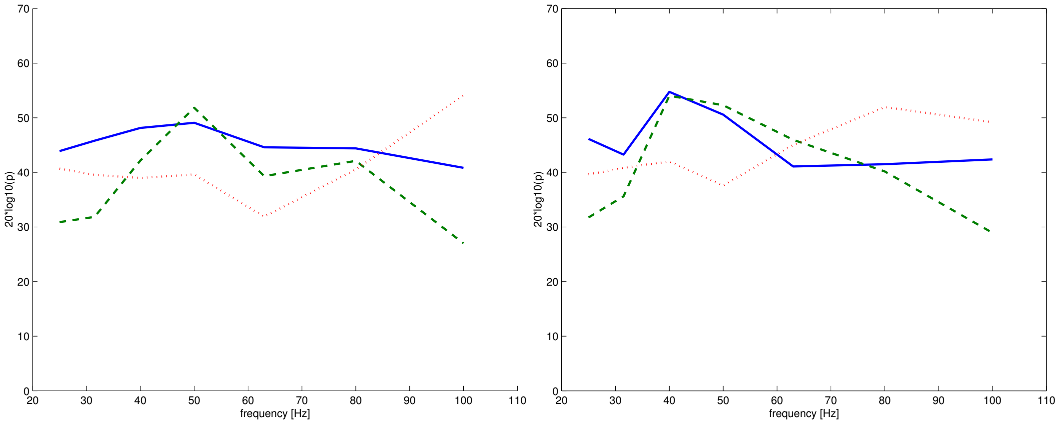
Another series of tests were carried out to examine the influence of the complexity of vibration distribution on the accuracy of the Rayleigh-based method. Vibration distributions of regular mode shapes ($n \times \lambda/2$) were applied to one selected wall of the test room. Calculations has shown that for most of the vibration distributions, after performing spatial and spectral averaging the Rayleigh integral based method gives very similar results to the results of the boundary element method (see Figure 3), but at certain mode shapes the differences are significant (see Figure 4). In the case when all 6 walls of the test rooms vibrated no such differences occurred.

4 Experimental validation

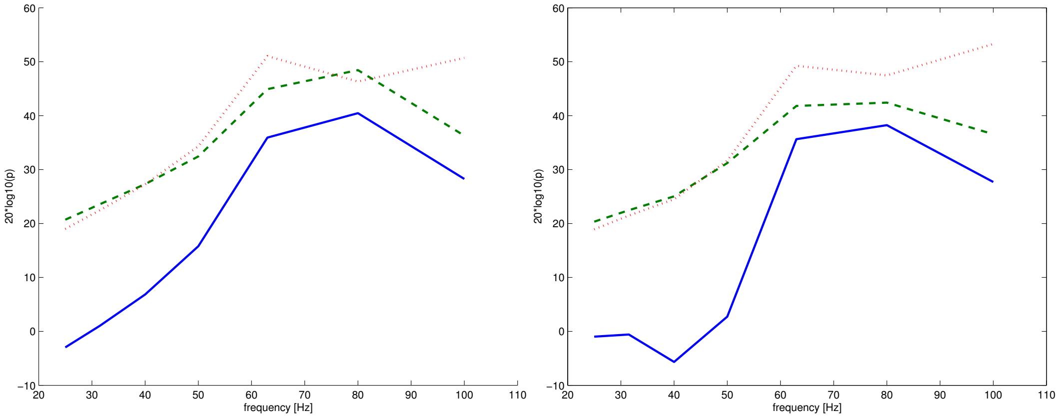
For the experimental validation of the newly developed technique, extensive measurement series was performed in a concrete building close to the tunnel of line B of the underground railway line network RER in Paris.

A room of 4.5 m x 5.8 m x 3.4 m was selected in the basement of the building with little furniture and clear concrete walls (see Figure 5). By using a parallel measurement system of 16 vibration and 6 noise channels more than 80 train passbys were recorded, out of which 62 were good enough to be evaluated. Sound pressures were measured in 6 points, vibration was measured altogether in 56 plus 1 reference point, providing a good quality data set suitable for model validations.

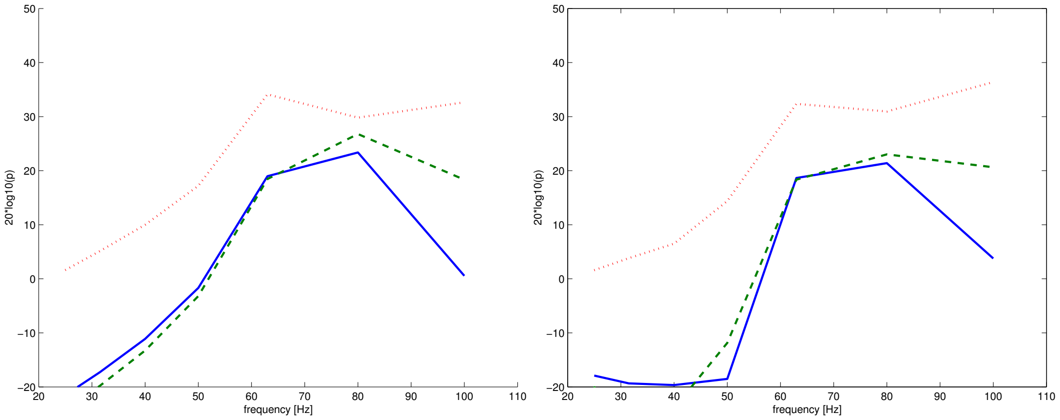
As input for the BE and Rayleigh-based calculations, a dense boundary mesh was needed with velocity values in each nodes of the mesh. Linear interpolation was used to get the vibration distribution along the



(a) Mode (1,1)



(b) Mode (2,2)



(c) Mode (4,4)

Figure 3: The influence of complexity of vibration distribution – Good agreement. Left: results of BEM calculations, right: results of Rayleigh-based method

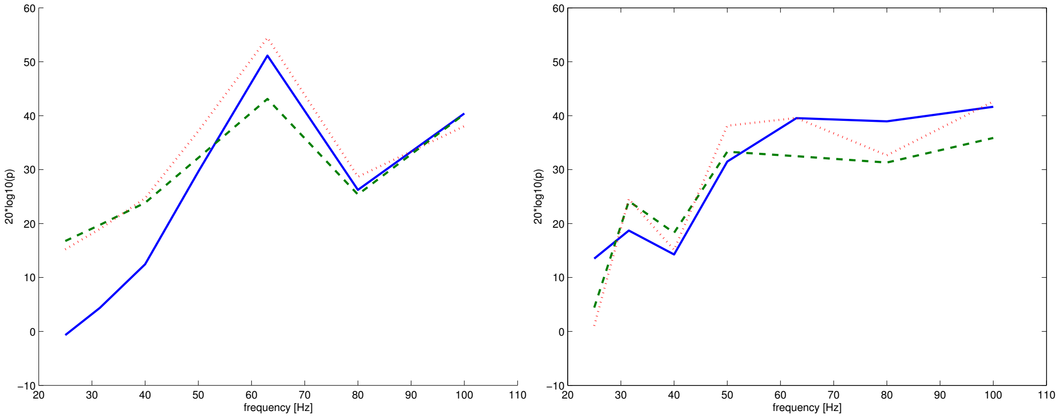


Figure 4: The influence of complexity of vibration distribution – Bad agreement at mode shape (3,2). Left: results of BEM calculations, right: results of Rayleigh-based method

Figure 5: The measurement site in the basement with accelerometers on the ceiling

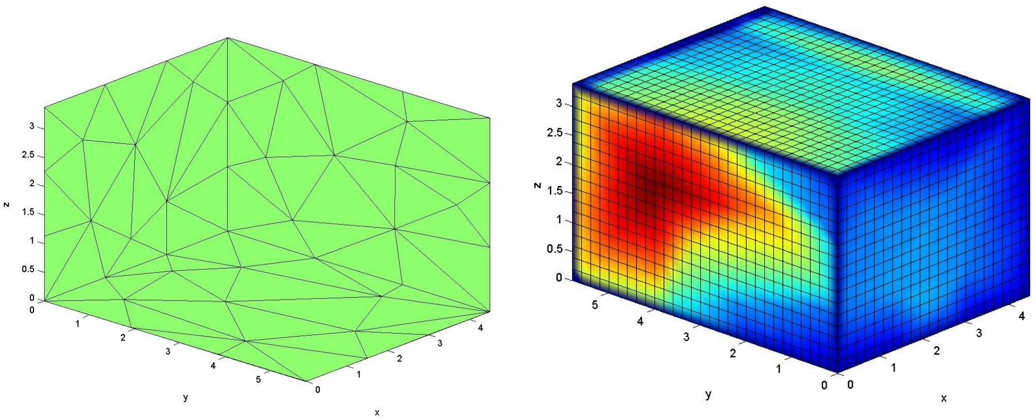
walls from the 57 measurement points. The mesh for interpolation can be seen in Figure 6(a), and the resulting boundary mesh representing the vibration values are shown in Figure 6(b). For the determination of the vibration distribution of the ceiling the results of experimental modal analysis were also used.

Figure 7 shows the results of boundary element modelling in one of the 6 microphone positions compared to the measured values, whilst figure 8 shows the results of the Rayleigh-based calculation (with 27 to 1331 total number of reflections taken into) in the same microphone position.

It can be seen that both methods give good approximation in tendency, but differences between the results can reach up to 8-10 decibels at certain frequencies. If we express the results in equivalent A-weighted levels the Rayleigh-based method's best result is a difference of 1.3 dB, the worst is 3.4 dB, whilst that of the BEM is 0.4 dB and 1.2 dB respectively.

5 The performance of the Rayleigh-based method, future plans

During the derivation of the Rayleigh integral based method several restrictions were made in order to get an easily implementable, robust method: calculations were restricted to shoebox shaped rooms allowing us to use a mirror image approach for handling sound reflections inside the enclosure without the need of visibility tests. On the other hand, this simplicity brought a serious disadvantage too, the number of image sources



(a) The interpolation mesh (b) The vibration distribution

Figure 6: The interpolation mesh and the vibration distribution used for calculations

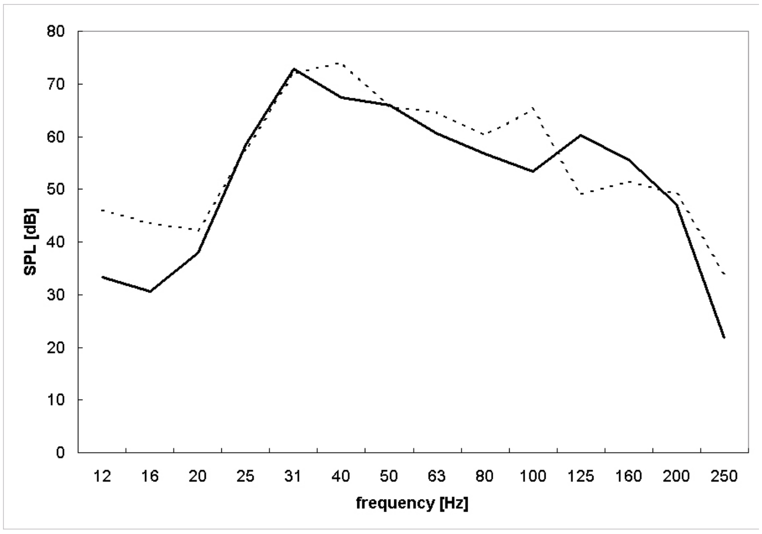


Figure 7: Results of the standard boundary element calculation compared to the measurement results. Dashed thin line shows calculated, solid thick line shows measured results

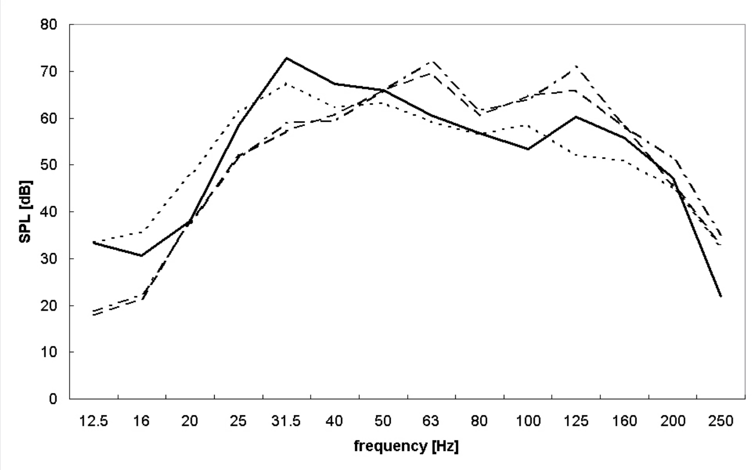


Figure 8: Results of Rayleigh-based calculations compared to the measurement results. Solid thick line shows measured results, dashed and dotted lines show results of Rayleigh-based calculation with different number of reflections taken into account

increases rapidly with the increasing reflection order. As the numerical tests and the experimental validation have shown, in a room with moderate absorption, good results can be achieved with relative few reflections.

However, in cases when more reflections is needed, eg. when the mean absorption of the room is too low, the Rayleigh-based method may slow down so much, that its reason of existence becomes uncertain in the light of the boundary element method.

As in room acoustics, using of a ray-tracing based method may solve this problem. The other main advantage of a ray-tracing based method would be the possibility of extending the applicability of the method to arbitrary shaped rooms without the need of visibility tests. It would also allow us using different absorption coefficients for each surface, making the prediction more accurate. Of course, it worths considering if such accuracy of the model is needed or not when results are highly averaged both in the spectral and spatial domains.

Another direction of development could be an even more statistical approach. After the first n reflections, the sound field can rather be characterised as reverberant. Thus after the first n reflections, the effect of the room (the boundary surfaces) can be taken into account as an additive constant depending on the reverberation time of the room. The task is to find the proper n for an acceptable error of prediction.

6 Conclusions

In this paper a newly designed, simple and robust method for predicting interior noise levels generated by underground rail traffic or other structural vibrations has been presented. The method is based on the Rayleigh integral and uses a modified Green's function, which gives the sound field in an arbitrary point in a shoebox shaped room, due to a point source placed on the boundary surface. Another assumption of the method is that acoustic absorption is evenly distributed along the walls of the room and can be described with a constant.

The method was verified by means of extensive numerical and experimental testing. The results of the tests were compared to the conventional collocational boundary element method.

According to these comparisons, it can be stated that the Rayleigh-based method gives reliable results, without the need of constructing and inverting large matrices, provided that both spatial and spectral averaging is used and the mean acoustic absorption of the room is not too low. The possible development directions of the method have been also outlined.

Acknowledgements

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