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Prediction of radiated noise in enclosures using a Rayleigh integral based technique

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Abstract [651] To predict radiated noisefield indoors we can use several methods. We can use a simple analytical calculation, for which we need radiation efficiency of the walls. The other well known method is the boundary element technique. But for both of them, we need highly detailed information on the vibration distribution on all of the walls. Generally, to determine this, one can use finite element modelling, that is a very computation intensive, and in some cases a practically not applicable task. This paper presents a newly developed simple method based on the Rayleigh integral that can be used to predict radiated noise in enclosures. The method is examined in different aspects – effect of absorption, effect of averaging in frequency and in space –, and compared to the other, well known technique, the boundary element method.

1 INTRODUCTION

Predicting interior noise levels in buildings generated by structural vibration is a typical task of the acoustic engineer. If the engineer has detailed information on the vibration distribution along the boundary surfaces, for example as a result of finite element analyses (FEA), the most accurate and most popular numerical way of predicting the radiated noise is using the boundary element method (BEM) [1, 2, 3]. The advantage of BEM is that it can be applied to arbitrary shaped enclosures and it can be easily connected to the FE modell of the building. The disadvantage of it is that BEM is a low-frequency method. In the case of a typical room, for calculations in the mid-frequency range the resolution of the boundary mesh should be increased and that would lead to the problem of solving a large system of linear equations. Inverting a fully populated matrix is very time consuming and despite the rapid development in the size of the memory modules, swapping would slow down the process even more.

The other possibility to predict reradiated noise is using analytical formulas. For this we need the radiation efficiency of the walls. Radiation efficiency varies rapidly in the low-frequency range and is hard to determine. These formulas are simple and thus give very rough results.

Rayleigh integral based methods are well known to solve exterior sound radiation problems. The technique presented in this paper is a new and simple way to predict interior noise levels. There is no need of knowledge of radiation efficiency, the method calculates directly from the radiation and geometrical data of the room. As compared to BEM, the matrix describing the phenomenon is much smaller and the inversion of it is also unnecessary.

2 DESCRIPTION OF THE RAYLEIGH INTEGRAL BASED METHOD

If we know the suface normal component of the vibration velocity distribution along an infinite vibrating plane ($v_n(\boldsymbol{r})$), we can use the Rayleigh integral to calculate the radiated sound pressure in an arbitrary point ($\boldsymbol{q}$) of the space [4, 5]:

$$p(\boldsymbol{q},\omega) = \iint_{\Sigma} g(\boldsymbol{r},\boldsymbol{q},\omega) v_n(\boldsymbol{r}) d\Sigma \tag{1}$$

where

$$g(\boldsymbol{r},\boldsymbol{q},\omega) = \frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r})}}{|\boldsymbol{q}-\boldsymbol{r}|} \tag{2}$$

is the Green's function of the homogeneous acoustical full-space, $k = \omega/c$ the wave number, ω is the circular frequency, and c is the speed of sound.

If we apply the Rayleigh integral to the case of closed spaces, we get a rough estimation of the radiated sound pressure. The result is the sum of sound pressures radiated by each wall to an acoustical half-space. Each wall is handled separately as part of an infinite plane. Only a small part (ie. the wall) of the infinite plane vibrates, the surface normal vibration velocity along this part is known, and $v_n = 0$ otherwise:

$$p(\boldsymbol{q},\omega) = \sum_{n=1}^{N} \iint_{\Sigma_n} g(\boldsymbol{r},\boldsymbol{q}) v_n(\boldsymbol{r}) d\Sigma \tag{3}$$

where N is the number of walls.

To take into account not only the primarily radiated sound but the reflections from the walls, we have to modify the Green's function (2). Each reflection can be handled as a direct sound arriving from a new sound source (an image source), the location of which is determined by mirroring the originally radiating part of wall to the reflecting walls, and the amplitude of which is reduced proportionally to the absorption at the reflections:

$$\hat{g}(\boldsymbol{r}_0,\boldsymbol{q},\omega) = \frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_0)}}{|\boldsymbol{q}-\boldsymbol{r}_0|} + \gamma_1(\omega)\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_1)}}{|\boldsymbol{q}-\boldsymbol{r}_1|} + \cdots + \gamma_i(\omega)\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_i)}}{|\boldsymbol{q}-\boldsymbol{r}_i|} + \ldots \tag{4}$$

where $\boldsymbol{r}_i$ is the location of the i-th image source and $\gamma_i(\omega)$ represents the effect of absorption.

This method is very complicated in the general case, for a large number of image sources has to be taken into account, and also their location and visibility has to be determined individually.

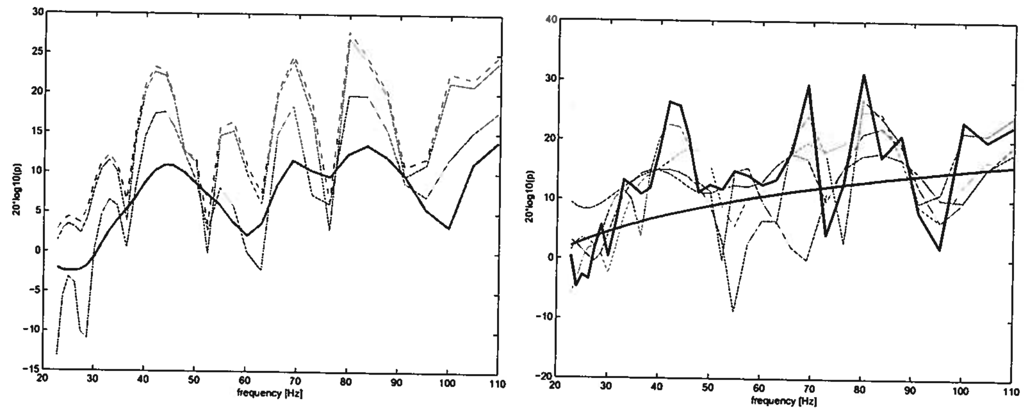
For simplicity, our investigations are restricted to the case of shoebox shaped rooms. This restriction enables us to neglect visibility tests and the determination of the location of the image sources can be easily automatized.

In the following we assume that the absorption of the walls is constant, and is represented by $\Gamma(\omega)$. With this we can convert (4) to the following form:

$$\hat{g}(\boldsymbol{r}_0,\boldsymbol{q},\omega) = \frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_0)}}{|\boldsymbol{q}-\boldsymbol{r}_0|} + \Gamma(\omega)\sum_{k=1}^{K_1}\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_k)}}{|\boldsymbol{q}-\boldsymbol{r}_k|} + \cdots + \Gamma(\omega)^m\sum_{k=1}^{K_m}\frac{e^{-jk(\boldsymbol{q}-\boldsymbol{r}_k)}}{|\boldsymbol{q}-\boldsymbol{r}_k|} \tag{5}$$

where K_i is the number of the i-th order reflections.

The surface integrals in equation (3) are performed numerically. The whole surface is divided into surface elements of constant velocity distribution. The integration of the Green's



(a) The effect of absorption: variations decrease with increasing absorption

(b) The effect of increasing the reflection order: resonant behaviour of the room can be observed

Figure 1: *The effect of absorption and reflection order on the Rayleigh-based calculations*

function over the elements can be carried out by means of Gaussian Quadrature integration. The discretized form of (3) is:

$$p(\boldsymbol{q},\omega) = \sum_{i=1}^{N_e} v_{n_i} \sum_{s=1}^{N_p} w_s \hat{g}(\boldsymbol{r}_{i,s},\boldsymbol{q},\omega) \tag{6}$$

where N_e is the number of elements, N_p is the number of Gaussian points, v_{n_i} is the surface normal velocity of the i-th element, w_s is the Gaussian weight and $\boldsymbol{r}_{i,s}$ is the location of the s-eth Gaussian point on the i-th element.

3 NUMERICAL TESTING RESULTS

This section decribes numerical tests which have been performed by means of modelling a shoebox shaped room of size 4 m × 5 m × 3 m in the x, y and z directions. The method of the numerical testing was to calculate the internal noise in different points by means of a direct BEM and the Rayleigh-based method, assuming a normal velocity distribution on the walls. The calculations have been performed in a frequency range: $f \in [10\text{ Hz}, 120\text{ Hz}]$, using $N_e \approx 1500$ surface elements for both methods.

3.1 Effect of absorption

To test the effect of absorption a simple case was examined, one surface element was chosen as the only vibrating wall section of the testroom. The sound field generated in the room by this one element was calculated with different absorption coefficients. The average absorption coeffeicient of the room was changed from 0 to 50 percent. The absorption coefficient gives the percent of sound energy absorbed at each reflection. The results of the investigations are shown in figure 1(a). The thin dashed blue line shows the case of 0 absorption and the thick solid purple line shows the case of 50 percent absorption, the other two lines show typical cases of absorption values between 0 and 50 percent. As expected the fluctuations decrease with increasing absorption.

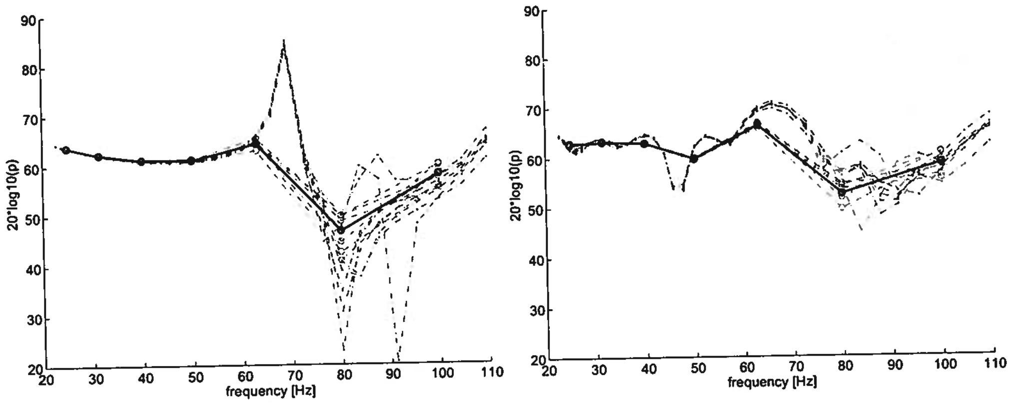


Figure 2: *The effect of spectral and spatial averaging on the noise spectrum for the case of the BEM and the Rayleigh-based calculations*

3.2 Effect of reflection order

The effect of reflection order was investigated as in the case of absorption. Absorption was set to 2% ($\Gamma = 0.99$). The number of reflections was increased from 0 up to 9261 (zero reflections mean applying the original Rayleigh integral to the walls).

The results can be seen in figure 1(b). The blue line shows the case of 0 reflections and the black line shows the case for 9261 reflections. Other curves are for appr. 30, 100, 350 and 1300 reflections. With increasing the number of reflections, the resonant behaviour can be observed more and more clearly.

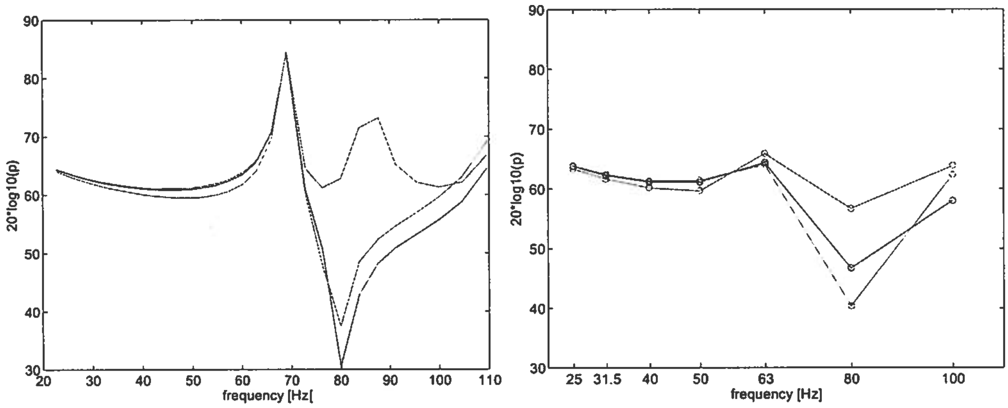
3.3 Averaging in the spectral and spatial domains

Figure 2 shows the narrow and third octave band spectra of noise levels calculated in seven points located closely around the internal point ($x = 1$ m, $y = 2$ m, $z = 1$ m), assuming constant velocity distribution all over the room's surface. The maximum distance between two points is 0.2 m. The blue curves show the narrow band spectra calculated in the seven points, the red curves show the third octave band spectra obtained by spectral averaging, and the black curve shows the third octave band spectrum averaged to the seven points. The left figure displays results calculated by the BEM, the right figure gives the Rayleigh method based calculations, taking a total number of 1331 reflections into account, and assuming an absorption coefficient of 2%, corresponding to $\Gamma = 0.99$.

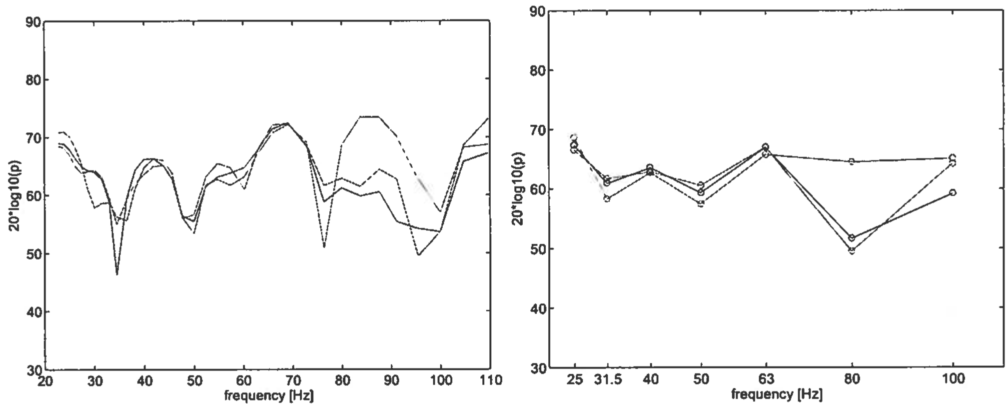
It can be clearly seen that although the narrow band specra can show significantly different levels (mostly at the peaks and dips), the smooth curves obtained by spectral averaging are similar for the case of the two methods. The necessity of spatial averaging is also shown.

Figure 3 shows narrow and third octave band spectra calculated in three points (averaged from 3×7 points) inside the enclosure, assuming constant surface velocity distribution, $\Gamma = 0.99$, and taking 1331 reflections into account. The results of the Rayleigh method are compared to those obtained by the BEM.

The curves in the figure describing the narrow band BEM results show two peaks, at approx. 70 Hz and 85 Hz. These peaks can be found in the narrow band spectra corresponding to the Rayleigh method, but the later figure contains additional peaks and dips in the lower frequency range. After the spectral averaging the curves are smoothed in booth figures, so the third octave band results of the two methods are similar in all three internal points.



(a) BEM



(b) Rayleigh method

Figure 3: *Narrow and third octave band noise spectra calculated by means of the BEM and the Rayleigh method in three points of the enclosure, assuming constant normal velocity distribution*

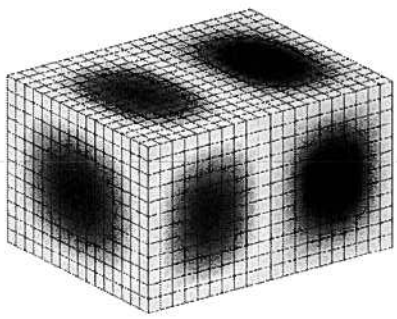
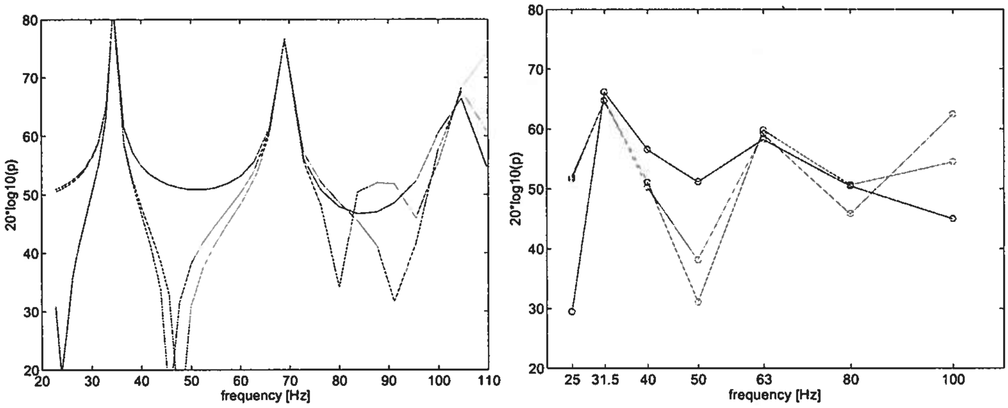
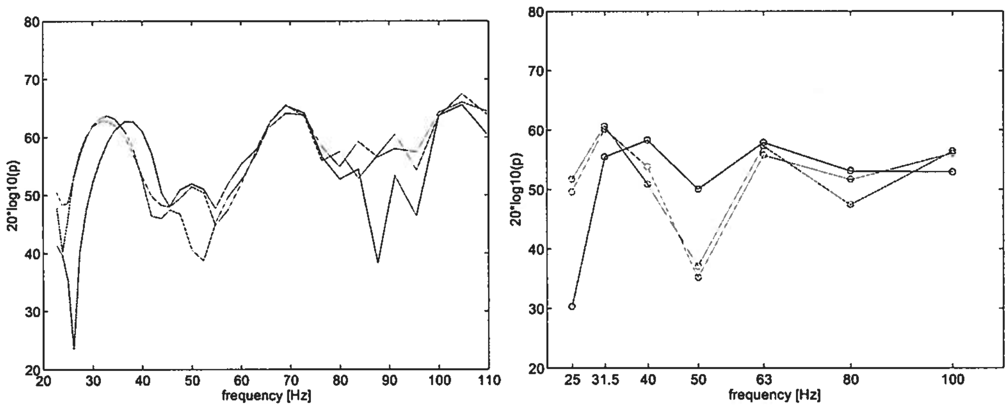


Figure 4: *Modal velocity distribution on the test-room*



(a) BEM



(b) Rayleigh method

Figure 5: *Narrow and third octave band noise spectra calculated by means of the BEM and the Rayleigh method in three points of the enclosure, assuming a modal velocity distribution*

Figure 5 displays similar results, but for the case of a more complex "modal" velocity distribution shown in figure 4. The modal behaviour of the room is clearly shown by the narrow band spectra calculated by the two methods, and the spectral averaging allows to get similar results not only in the modal frequencies, but also in the noise levels.

4 CONCLUSION

In this paper we have presented a newly designed simple method for predicting radiated noise levels in enclosures. The method is based on the Rayleigh integral combined with a modified Green's function which gives the sound field in an arbitrary point in a shoebox shaped room, due to a point source placed on the boundary surface.

Investigations have shown the effects of (1) absorption of the walls, (2) the number of reflections taken into account and (3) spectral and spatial averaging.

According to comparison to the result obtained by the boundary element method, it can be stated that the Rayleigh-based method gives reliable results without the disadvantage of creating and inverting large system matrices.

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