

## DEVELOPMENT OF AN INVERSE BOUNDARY ELEMENT TECHNIQUE FOR PARTIAL NOISE SOURCE IDENTIFICATION OF TIRES

Fülöp Augusztinovicz, Ferenc Márki and János Granát<sup>1</sup>  
Wim Hendricx and H. Van der Auweraer<sup>2</sup>

1 Technical Univ. of Budapest, Dept. of Telecommunications, H-1111 Budapest, Sztoczek u. 2.

2 LMS International, B-3001 Leuven, Interleuvenlaan 68.

### INTRODUCTION

The identification of noise generation mechanisms inherent in tire/road interaction phenomena requires sophisticated instrumentation and measuring techniques. Due to the nature of the problem, the application of conventional vibration sensors is largely limited, hence those methods making use of acoustic sensing are of primary importance. Nearfield Acoustic Holography (NAH, [1, 2]) is one of these techniques. Other useful techniques are, among others, the Airborne Source Quantification (ASQ) method [3] and a whole group of inverse FRF methods [4,5], originally developed for excitation force identification in pure mechanical systems [6].

This paper reports on an inverse FRF method which makes use of numerically calculated transfer functions between the radiating (or source) and sensing (or measurement/holography) surface. The technique, originally proposed in this form by Mas *et al.* [7], can be considered as a generalization of the AH technique. Unlike NAH though, it is not burdened by the limitation that both the source and the measurement surface must be plane or of some other elementary shape, which is of vital importance from the tire analyst's point of view. Its close relationship with the Boundary Element Method implies that it is usually denoted as an inverse BE method (I-BEM).

### SUMMARY OF THE UNDERLYING THEORY

The inverse BEM method is based on the discrete form of the governing equation of a general radiation problem

$$[A]\{p_s(x)\} - [B]\{v_s(x)\} = \{p(y)\} \quad (1)$$

relating the sound pressure  $p_s$  and particle velocity  $v_s$  in any arbitrary node  $x$  along the source surface mesh to any arbitrary point  $y$  outside of the surface through the influence matrices  $[A]$  and  $[B]$ . Solving the problem by using the collocational method, the pressure  $p_m$  to be measured along the measurement plane, usually selected in the vicinity of the source, can be expressed as

$$\{p_m\} = \left[ [a] [A]^{-1} [B] + [b] \right] \{v_s\} = [c] \{v_s\} \quad (2)$$

The matrix  $[c]$  is referred to as the transfer matrix of the system and  $\{v_s\}$  is the vector of sought surface velocities. The solution of Eq. (2) is in principle rather straightforward, provided that the

number of microphone positions along the measurement surface is greater than, or equal to, the number of elements on the source surface, and that the matrix  $[c]$  is not singular.

Note that a more detailed derivation of the technique can be found in [9].

## LIMITATIONS AND IMPROVEMENT POSSIBILITIES OF THE I-BEM

It is known for all inverse methods that the accuracy of the method largely depends on the condition number of the transfer matrix, in this case  $[c]$ . As the condition number of the matrix increases, the obtained result  $\{v_s\}$  is increasingly influenced by modeling and measurement errors. It was shown for plane measurement arrays analytically and experimentally [5], how the sensitivity to errors can optimally be controlled by selection of appropriate measurement points. We have performed simulation calculations with more complex measurement surfaces around various source models. It was established that those simple rules developed by Dumbacher *et al.* (equal source/measurement surface spacing, source and microphone points in line, source-microphone distance less than source point spacing) do hold for a wider variety of models and surfaces as well.

In some practical cases not all of these rules can be followed exactly. If the condition number of  $[c]$  increases, a simple overdetermination of Eq. (2) and a standard least mean square solution is not sufficient to obtain meaningful results. In order to limit the computation error to practicable levels, some kind of regularization methods shall be resorted to. Two methods have been investigated: Tikhonov regularization and Truncated Singular Value Decomposition (TSVD) [8]. We have obtained better results for the solution of the I-BEM problem by using the TSVD method. By increasing the extent of regularization, the solution becomes less sensitive to modeling/measurement errors, at a price of decreased spatial resolution.

## PRACTICAL IMPLEMENTATION FOR TIRE NOISE INVESTIGATIONS

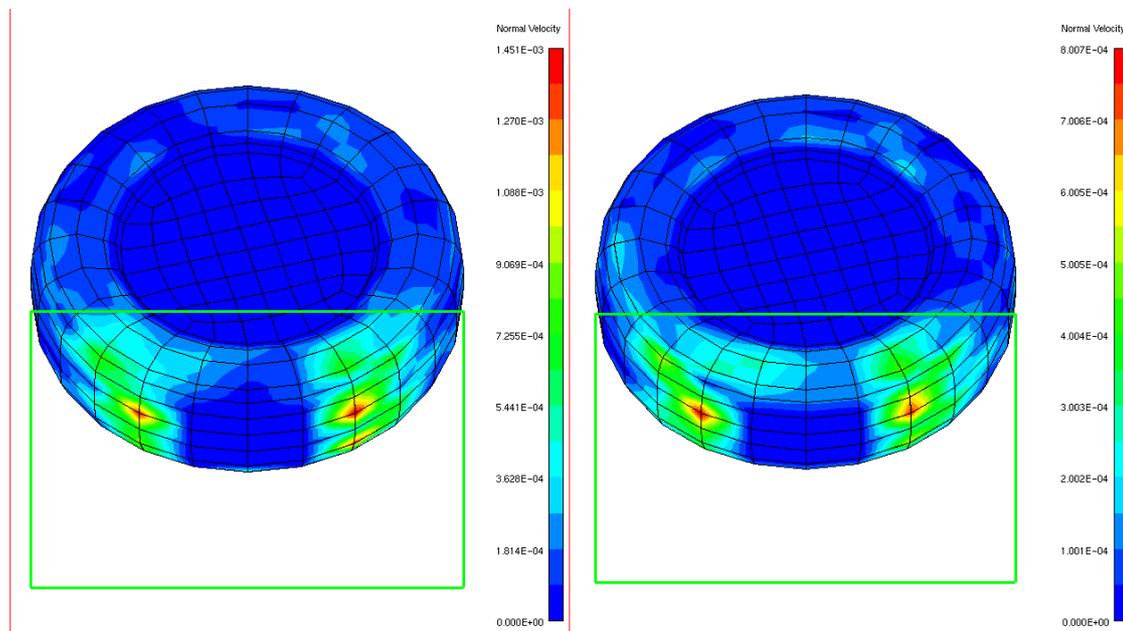
The practical implementation of the method is largely based on the vibroacoustic prediction software package SYSNOISE, embedded in MATLAB environment. Following the aforementioned rules, an appropriately matched discretized source model / measurement surface is to be defined first. The sound pressure field is scanned by a microphone array and the data processed by a multichannel measurement system. The influence and contribution matrices are calculated by SYSNOISE. Both the measured sound pressures and the required numerical matrices are imported into MATLAB through small interface programs. Eq. (2) is solved by means of a MATLAB program package [8] and the obtained results are eventually exported to SYSNOISE for visualization.

After having tested the method on small testing objects, the procedure was applied on the tire noise problem. First a wooden mock-up with six built-in, independent loudspeakers was constructed and calibrated by means of a laser Doppler vibrometer. The mock-up was then placed in an anechoic chamber and the microphone array measurements repeated twice: with the mock-up placed app. 1.2 m above the absorbent floor and directly on a wooden floor plate, simulating road reflections. The obtained source velocity distribution is given in the table (see next page).

As one can see, the obtained results are promising. Not only the radiating surfaces can be easily identified, but the measured results are correct in qualitative sense as well. It is worth noting that reasonable estimations could also be obtained for the reflective case in the vicinity of the contact patch, where the acoustic environment of the partial source is rather adverse.

Loudspeaker	Measured and calculated volume velocities of the various loudspeakers for the non-reflective case [ $*100 \text{ cm}^3/\text{s}$ ]					
	220 Hz		480 Hz		700 Hz	
A	161.9	187.9	62.9	73.1	48.6	61.1
B	54.0	58.3	20.7	22.9	12.6	13.9
C	152.9	182.5	63.8	68.8	42.3	46.7
D	36.0	43.0	16.2	18.7	10.8	15.0
E	152.9	171.4	58.4	64.7	41.4	40.7
F	152.9	172.6	71.9	77.9	44.1	48.9

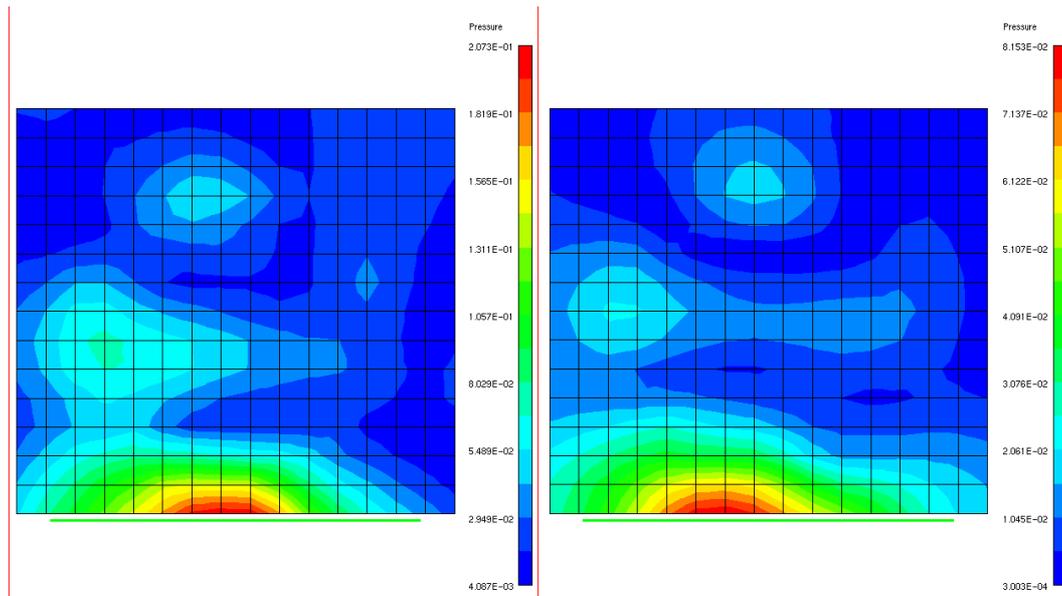
The I-BEM method was also applied on real tyre data. A set of measurements performed by LMS International and Pirelli, aimed originally at using for ASQ computations, were used. 15 microphones were placed along a U-shape microphone holder, which was then turned around the wheel running on a test drum in an anechoic chamber. The model mesh consisted of 288 nodes, appropriately chosen to the 336 microphone positions in which the pressure measurements were made. From the point of view of I-BEM, not all of the microphone positions were in the most favourable positions and 8 positions were missed, therefore the generated transfer matrix had a large condition number ( $\sim 10^7$ ).



**Fig. 1.** Calculated velocity distribution along a tyre at 722 Hz with TSVD (left) versus Tikhonov (right) regularisation

In the first step, the source surface velocity along the tyre was computed by using I-BEM. Thereafter a simple radiating calculation procedure was also performed for a plane hologram surface, where pressure values measured for other purposes were also available. The computed and the measured pressures along the hologram plane can be seen and compared on figures 9 and 11, the calculated source velocity distributions on figures 8 and 10.

We have established that for low frequencies the choice of regularisation method is critical, different choices may give radically different results. This is not the case for higher frequencies (see Fig. 1. For 722 Hz), where both regularisation methods (TSVD and Tikhonov) gave similar results.



**Fig. 2.** Comparison of sound pressure fields of the tire at 722 Hz: direct measurement (left) and I-BEM calculation (right)

## CONCLUSION

The inverse BE method seems to be a viable alternative of NAH in those cases when the source is of complex shape. The obtained results enable the analyst to draw not only qualitative but quantitative conclusions too. In order to get meaningful results, an optimally matched source model / measurement point mesh surface, placed close to the source, is essential. Even if this is the case, regularization methods to solve the inverse problem is usually required.

## ACKNOWLEDGEMENT

The work reported herein has been performed in the framework of the BRITE-Euram Project TINO. The Hungarian participation in the project was funded by the Hungarian National Committee for Research and Development under the contract EU-98-B4-104. Special thank is also due to PCB International for supporting the Laboratory of Vibroacoustics of TU Budapest.

## REFERENCES

- [1] Maynard, J.D., Williams, E.G. and Lee, Y.: „Nearfield acoustic holography: I. Theory of generalized holography and the development of NAH”. JASA, 78 (4) pp. 1395-1413. (1985)
- [2] Hald, J.: STSF – a unique technique for scan-based nearfield acoustic holography without restrictions on coherence. B&K Technical Review, 1988.
- [3] P.van der Linden and D. Otte, „Diagnosis of Pass-by Noise of road vehicles”. 4<sup>th</sup> ATA Conf., pp. 1268-1276.
- [4] Dumbacher, S.M. and Brown, D.L., „Source imaging using acoustic inverse FRF array technique”. 14<sup>th</sup> Int. Modal Analysis Conf., Dearborn, Vol. II. pp. 1322-1331 (1996)
- [5] Dumbacher, S.M. and Brown D.L., „Evaluation of noise source ID using analytical FRFs”. Proc. 16<sup>th</sup> Int. Modal Analysis Conf., Santa Barbara, Vol. II. pp. 1292-1302 (1998)
- [6] Dobson, B.J. and Rider, E: „A review of the indirect calculation of excitation forces from measured structural response data”. Proc. IME, Vol. 204, pp. 69-75 (1990)
- [7] Mas, P. and Sas, P. and Arruda, J.R.F.: “Minimization of statistical and deterministic errors in nearfield acoustic source identification”. Proc. Inter-Noise 1997, Budapest, Vol. III. pp. 1295-1299 (1997)
- [8] Hansen, P.C, „A MATLAB package for analysis and solution of discrete ill-posed problems”. Technical University of Denmark, 1992. Available via Netlib ([netlib@research.att.com](mailto:netlib@research.att.com))
- [9] F. Augusztinovicz, F. Márki, J. Granát, W. Hendricx, H.V.d. Auweraer, „Extension of acoustic holography for tire noise investigations”, CD Rom of 137<sup>th</sup> Meeting of ASA/2<sup>nd</sup> Forum Acusticum, Berlin, 1999.