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TORSIONAL MODAL ANALYSIS OF SINGLE SHAFT SYSTEMS BY USING A NOVEL VIBROACOUSTIC MODEL OF A PERMANENT MAGNET SYNCHRONOUS MOTOR

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Abstract: In this paper a novel torsional experimental modal analysis method based on a new vibroacoustic modeling methodology of a permanent magnet synchronous motor (PMSM) is presented. This methodology accurately describes torsional vibration conditions of a PMSM in case of single-phase pulsating current excitation, in conjunction with small angular displacement of the rotor. The test rig and the modal analysis results are briefly outlined, and an experimental validation consisting of a torque time-frequency (signature) analysis is introduced. The main advantage of the proposed method is that it does not require an external broadband torque source; since the PMSM is part of the investigated system.

Keywords: Torsional vibration, Modal analysis, Rotor dynamics, Brushless permanent magnet synchronous machine

1. Introduction

The motivation of this current research came from two different industrial fields. On one hand, the growing demand in the automotive industry for high performance DC motor drives (fuel/water pumps, power-assisted steering systems, engine cooling fans and the like) is more and more satisfied using permanent magnet synchronous motor (PMSM) drives instead of standard DC motor drives, due to their higher reliability, and lifetime, lower maintenance cost and lower electromagnetic interference (EMI). On the other hand, in the design, fabrication and diagnostics of rotating machinery, turbines, pumps, vehicle power-trains and the automotive DC drives mentioned above, analytical and experimental torsional modal analysis are fundamental and widely used methods.

In this paper a novel torsional excitation technique is introduced that uses the PMSM in the investigated system to produce adequate broadband torque excitation, and a very simple, however rather obvious accelerometer placement method to generate the angular acceleration signals is shown. Using the torque input and angular acceleration output, an experimental modal analysis on a single shaft torsional system, consisting of a PMSM, a coupler, a torque sensor, and a high power asynchronous motor was successfully performed.

Eventually, the measured natural frequencies were validated by using a standard signature analysis of the torque signal being measured during a motor run-up.

2. Background

In this chapter the modal analysis techniques of torsional systems, the measurement and excitation of torsional vibrations are briefly summarized. Finally, based on the literature, standard modeling of PMSMs is presented.

2.1. Modal analysis techniques in torsional systems

In the solution of torsional vibration based problems one of the most often applied approach is - the analytical, numerical, or experimental determination of natural frequencies and mode shapes of a system - modal analysis in brief.

There are hundreds of case studies in the literature about machine failures caused by a resonance, i.e. some periodic excitation - for example an order of a combustion engine - hitting a torsional natural frequency and resulting in unbearable high stress in a component (shaft, gear, blade) leading to machine failure [1], [2], [3].

In order to solve torsional vibration related problems it is advisable to calculate the natural frequencies of the system (even with a simple mass-spring model, or using a more sophisticated finite element method analysis [1], [4]), and after the experimental validation of the calculation, the mathematical model can be used to predict the effect of structural modification and the changes of natural frequencies.

If problematic natural frequencies cannot be shifted sufficiently or the equipment is working in a wider speed range, forcing functions have to be taken into account and the forced response and stresses have to be evaluated. If the calculated stresses exceed the safe stress limit, the damping of resonant peaks or more extensive structural modification can solve the vibration problem.

If the torsional system cannot be excited by a known input directly, operational deflection shape analysis or, in other words, output only modal analysis can be a way of orientation about the system's behavior. The results of this method are deflection shapes of the system under operating conditions, thus describing the vibration conditions under normal workload. In this way the normal modes can be measured in resonance condition only, when one excitation component hits a natural frequency. Even though this method is feasible, in higher power applications the machine cannot be run in a resonant state for a long time.

2.2. Measurement of torsional vibration

The torsional vibration of an evenly rotating structure is the small change of angular velocity. Until the advent of laser vibrometers the measuring method of this small change was the comparison of the average angular velocity to the instantaneous angular velocity, and converting the result into an easily measurable physical quantity, usually to voltage.

One of the most simple - but still applied - approaches is the use of a seismic mass mounted resiliently to the rotating object. The inertia of the seismic mass and the stiffness of the mounting produce a mechanical low-pass filter, which - beyond its cut-off frequency - removes the fast changes from the angular velocity, hence providing a reference angular velocity. Measuring the difference between these two angular velocities the torsional vibration can be determined.

One of the first equipment utilizing this principle was patented in 1940 under the name 'optical torsiograph' and used slotted disks mounted on the rotating shaft and the seismic mass, and a light bulb to produce a light beam passing through the slots and trace out the torsional vibration pattern [5]. In 1941 C. S. Draper patented equipment utilizing an electrodynamic transducer to measure the difference between the two angular velocities and produce an output voltage proportional to the torsional vibration [6]. In the end of the 1950's F. E. Booth constructed a torsional vibrometer according to the seismic mass principle, however - in order to achieve high accuracy - with capacitive sensor [7].

In the second half of the 1940's another equipment - also called torsiograph - appeared, but it was based on a different principle. A toothed wheel was attached to the rotating shaft, placed in a non-rotating U shape iron core, thus varying the magnetic resistance of this magnetic circuit. The winding on the iron core was excited by direct current, so an almost sinusoidal alternating current was induced in the coil according to the number of teeth and the speed of rotation. In this scheme the frequency variation of the output voltage is proportionate to the torsional vibration, and a special apparatus was designed to trace the vibration waveform on an oscilloscope [8].

In 1975 John F. Wolfinger simplified the magnetic pickup, and came to the great idea that the frequency variation in the output signal of the magnetic pickup is in effect frequency modulation, therefore, it could be demodulated by using a standard FM demodulator [9]. This invention suggests a phase locked loop (PLL) circuit to produce the necessary output signal. This is in complete analogy with the seismic mass based methods, because the frequency of the PLL's local oscillator plays the same role as the angular velocity of the seismic mass, and the phase detector converts the frequency difference to voltage that is proportional to torsional vibration.

Several inventions aimed at refining the electronics of the previously described equipment [10], [11], and a number of suggestions referred to the change of the magnetic pickup to a permanent magnet generator [12], [13].

The toothed gear approach is still widely used in the industry. As a recent example, LMS International has developed a four channel torsional vibration measurement module that converts the inductive pickups' signals into a bit stream, and processes it in a DSP to calculate the torsional vibration [14].

Torsiographs based on the seismic mass principle are also widely used in the industry, for instance HBM manufactured a sensor that is frequently mentioned in case studies [1].

Another - but rather expensive - way of measuring torsional vibration is making use of laser vibrometry. The theory behind torsional laser vibrometers is the following: two independent, strictly parallel laser beam scatters on the rotating shaft, and the reflected light is processed in two interferometers to get the Doppler shift - that is proportionate to the surface velocity - of each beam. It can be shown that the velocity difference of the two measurement points is only proportionate to the angular velocity of the shaft, hence providing good immunity against lateral vibration. In this way a very versatile and comfortable instrument can be designed and the benefit of non-contact, fast and accurate measurement is substantial. Instruments based on the principle outlined above can be purchased from prestigious manufacturers such as B&K [15] and PolyTec [16].

2.3. Excitation techniques of torsional vibration

Already in the 1930's equipment was designed to test transmissions, rear axles, differentials, gearboxes, and other automotive units under nominal torque and speed conditions. Nevertheless, these test stands could not provide torsional vibration load on the targets [17]. The first equipment that able to produce sufficient torsional vibration has been patented by A. M. Dudley in 1945. The main goal of the invention was to test aircraft propellers without a full combustion engine but with an electric drive [18]. To test propellers on its natural frequencies L. A. Kilgore developed a similar equipment implementing a feedback loop with gaseous discharge tubes (Thyratrons) [19].

Similar testing machines were developed in these times. Gerald S. Zobrist gave a good comprehension in his patent about this equipment and presented a torsional vibration generator, built up with a hydraulic motor [20]. This instrument was able to provide excitation for modal analysis purposes too.

In 1996 William T. Gaddis presented a torsional exciter for turbine propellers in which a stack of piezoelectric crystals were used to produce torsional vibration, so a new technology appeared in the field of torsional exciters [21].

Christof Sihler published a method and equipment to produce a pulsating torque component in shaft assemblies with large flywheel generators (synchronous machines), developed especially for modal testing purpose [22].

Naturally, several pieces of equipment were developed to provide well controllable and stable torsional vibration for calibration purposes of the measurement instrumentation [23], [24], [25].

2.4. Modeling a PMSM

Fig. 1 shows a schematic section view of a PMSM and its circuit diagram.

The electrical behavior of permanent magnet synchronous motor can be summarized in the following equations [26]

$$u_{si} - u_0 = R_s i_{si} + L_s \frac{di_{si}}{dt} + u_i \,, \tag{1}$$

$$\sum i_{si} = 0\,, \tag{2}$$

where $i = 1,2,3$, and u_{si}, i_{si} are the stator voltage and current of the i^{th} phase, u_i, u_0 are the induced voltage of the phases and the voltage of the common end of the windings (see *Fig. 1b*), and R_s, L_s are the resistance and inductance of the phases respectively. This model presumes that the rotor-position dependence of the inductances and the coupling between windings is negligible. In most cases - for instance in the design of a closed loop control - this assumption is acceptable.

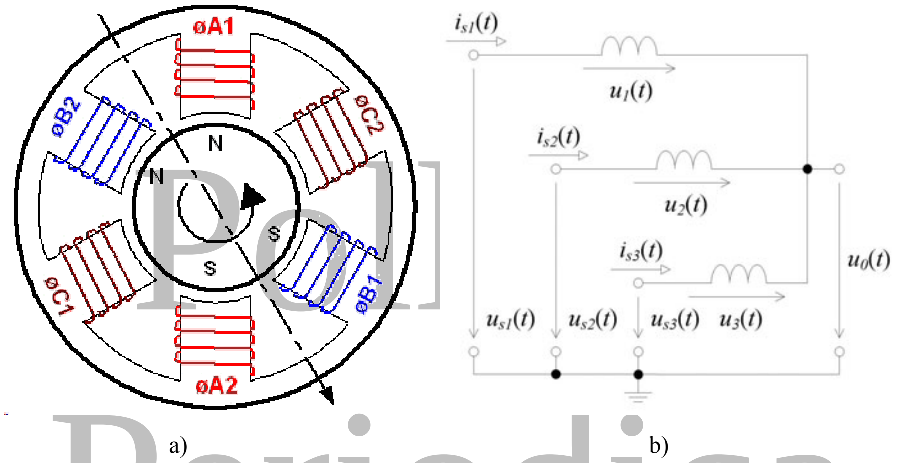


Fig. 1. a) A schematic section view of a PMSM, b) The schematic circuit diagram of a PMSM with wye-connected phases

The induced voltage can be expressed from the flux change

$$u_i = \frac{d\psi_{ir}}{dt} = \frac{\partial \psi_{ir}}{\partial \varphi}\frac{d\varphi}{dt} = \theta \frac{\partial \psi_{ir}}{\partial \varphi}\,, \tag{3}$$

where ψ_{ir} is the rotor flux in the i^{th} winding, φ is the angular displacement, and θ is the angular velocity. The torque acting on the rotor can be calculated from the phase currents and from the flux

$$m = \sum_{i=1}^{3} m_i = \sum_{i=1}^{3} i_{si} \frac{\partial \psi_{ir}}{\partial \varphi}\,. \tag{4}$$

The structural dynamics of the motor is described by the following two equations

$$\begin{aligned} &\frac{d\varphi}{dt} = \theta, \\ &\frac{d\theta}{dt} = \frac{1}{J_r}\left(m - m_l - m_c - B\theta - C\operatorname{sign}(\theta)\right), \end{aligned} \tag{5}$$

where J_r is the moment of inertia of the rotor, m_l is the load torque, m_c is the cogging torque, B is the viscous friction coefficient, and C is the Coulomb friction coefficient.

The cogging torque is due to the attraction of the permanent magnets and to the salient pole pieces of the stator iron, therefore the cogging torque is always present, even in the absence of the phase currents.

It is common in practice to express non-sinusoidal quantities ψ_{ir} and m_c by their Fourier series; however, calculations in section 3 do not require these details.

3. Using the vibroacoustic model in torsional EMA

Experimental modal analysis (EMA) is a measurement-based procedure that can determine the natural frequencies and mode shapes of a mechanical system.

Basically, for experimental modal analysis a number of frequency response functions (FRFs) between several input and output points of the system are required. The excitation should be force, and the response can be displacement, velocity or acceleration. In case of a torsional system excitation should be torque and the response can be angular displacement, angular velocity, or angular acceleration [4].

In this case, vibroacoustic means that the rotor does not whirl but rather oscillates around an equilibrium point, so it vibrates torsionally. A reason for that can be an applied non-whirling magnetic field, i.e. when the excitation is not three-phase, and/or broadband (for instance white noise).

Without going into details, the most important considerations necessary to understand the theory behind this new modal analysis method are outlined, [27] contains further information about vibroacoustic modeling a PMSM.

Equation (4) gives guidance on torque generation. Around an equilibrium point the derivative of the flux can be substituted with a position-dependent constant

$$m = i_{si}\frac{\partial \psi_{ir}}{\partial \varphi} = i_{si}T(\varphi)\,. \tag{6}$$

This means that well-defined torque excitation can be generated by using one-phase current excitation. Based on (1) this is possible by using a current source because a current source does not determine its terminal voltage, so the voltage across R_s, L_s, and the induced voltage do not affect the current flowing through the winding.

The dynamics of the motor does not affect the modal analysis procedure because the motor itself is part of the system to be analyzed. The only phenomenon that should be treated is the vibroacoustic effect of the cogging torque.

Writing the cogging torque as an arbitrary function of φ

$$m_c = m_{cg}(\varphi), \;\; [\mathrm{Nm}], \tag{7}$$

and expressing the derivative of this function

$$\frac{dm_c}{d\varphi} = \frac{dm_{cg}(\varphi)}{d\varphi}, \;\; [\frac{\mathrm{Nm}}{\mathrm{rad}}]. \tag{8}$$

This derivative has a torsional stiffness dimension, therefore, the cogging torque can be taken into account as torsional spring acting on the motor shaft.

3.1. The experimental setup

The single shaft system containing a PMSM can be seen in *Fig. 2*. This experimental setup is used in the testing of PMSMs for power assisted steering systems. The components of the test rig are the following: 1. PMSM; 2. Accelerometer; 3. Clutch; 4. Coupling; 5. Torque sensor; 6. Clutch; 7. Load motor.

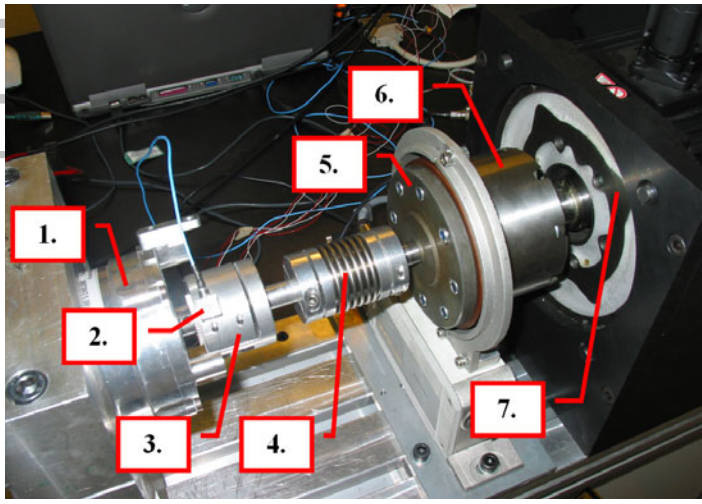


Fig. 2. The analyzed shaft assembly containing a PMSM

The PMSM was driven by means of a dedicated audio power amplifier with white noise current output. Data (current, acceleration, torque) acquisition and white noise generation was performed by means of an LMS SCADAS III. multichannel front-end processor, and a standard laptop running LMS Test.Lab software. Sampling frequency was 8192 Hz. Modal analysis was carried out in Test.Lab, and the spectral analysis was implemented in MATLAB.

Special attention has to be taken on the simplicity of the proposed modal analysis technique, hence the measurement of the response was done by using a standard triaxial accelerometer, but with a tricky positioning. Only the tangential component of the acceleration has to be recorded and divided by the distance between the accelerometer's measuring axis and the axis of the shaft, resulting in the required angular acceleration. Of course instead of a triaxial accelerometer a standard single axis accelerometer can be used, if it can be mounted in the tangential direction.

The measurement points and the wire frame model of the shaft assembly are shown in *Fig. 3*.

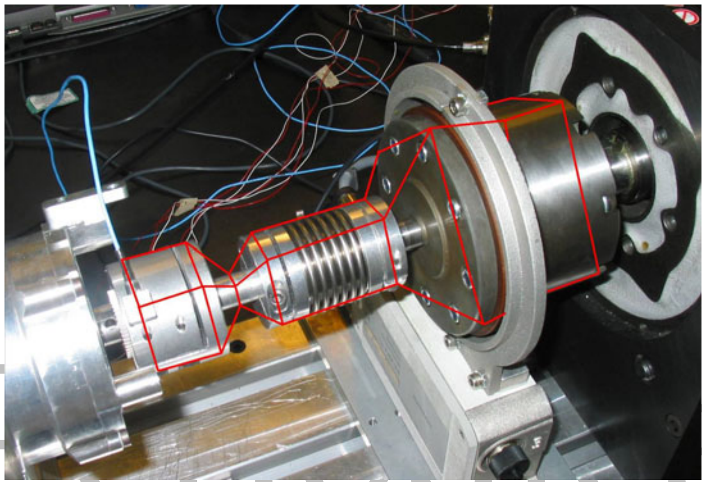


Fig. 3. The measurement points and the wire frame model of the shaft assembly

3.2. Measured mode shapes

The first mode (47 Hz) of the shaft assembly is shown in *Fig 4*. The almost negligible deformation is well observable; the whole torsional system is vibrating as a rigid body on the spring originating from the cogging torque. The relatively high natural frequency of this mode should also be mentioned.

Fig. 5 shows the shape of the first normal mode (298 Hz), and the following unexpected phenomenon can be observed: this mode is basically the result of a simple mass-spring resonant system, where the mass is the moment of inertia of the rotor, and the spring is the whole shaft assembly between the torque sensor and the rotor.

The consequence is that the natural frequency of this mode shape - which is in the frequency range of the torque measurement - cannot be shifted significantly by using a changed coupling, because every piece of this shaft section is equally deformed. Therefore, a dramatic change in the natural frequency would need a dramatic change in the coupling stiffness.

Fig. 6 shows the last mode (695 Hz) in the frequency range of interest. In this mode shape the torque sensor plays a key role due to its stiffness stemming from its working principle.

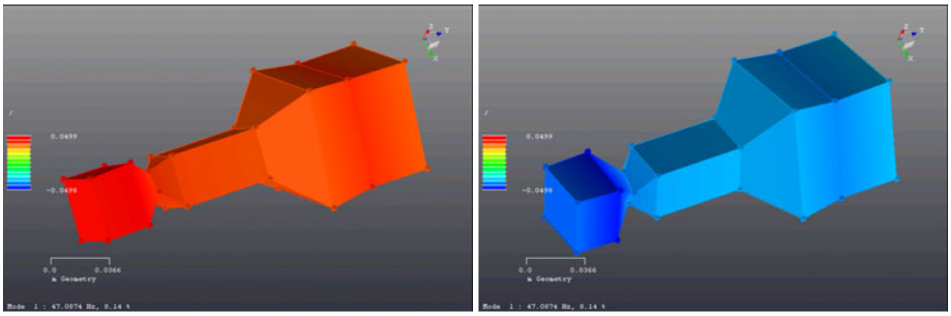


Fig. 4. The 1st mode of the shaft assembly at 47 Hz

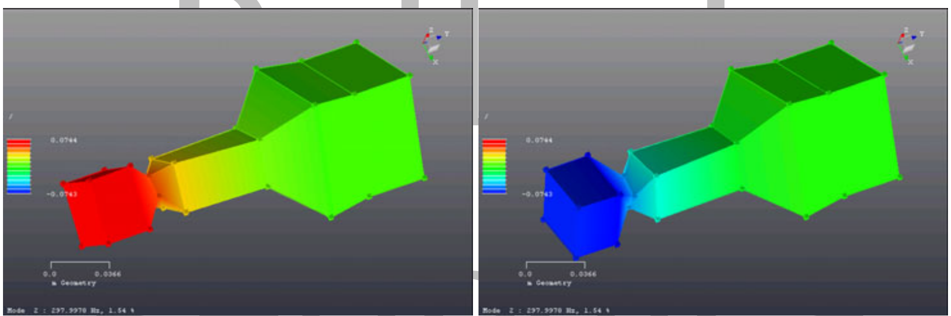


Fig. 5. The 2nd mode of the shaft assembly 298 Hz

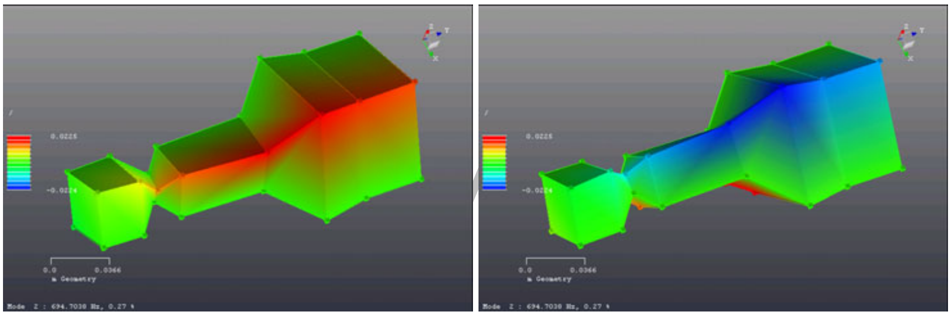
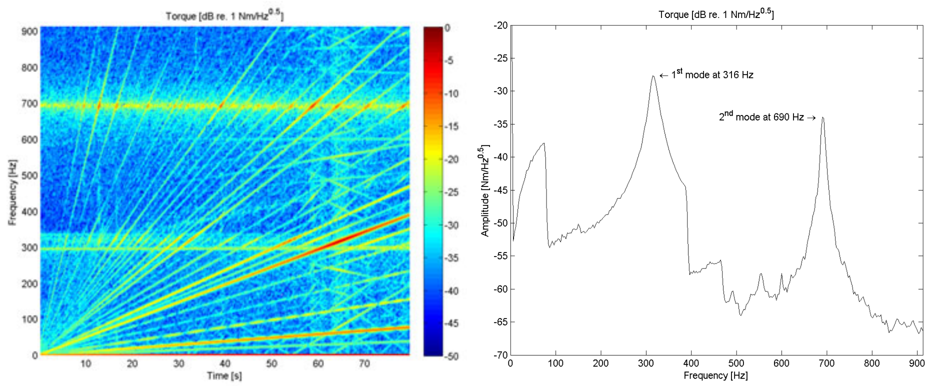


Fig. 6. The 3rd mode of the shaft assembly 695 Hz

4. EMA results validation by means of signature analysis

In order to validate and prove the existence of the measured mode shapes, a torque signature analysis was carried out on the test rig. The speed of the motor was increased from 0 to 2400 rpm in 80 s, and the load of the motor consisted of three high power wye-connected 1 Ω resistors. The Campbell plot (which is also called as spectrogram, order plot, or signature) of the recorded torque signal is shown in *Fig. 7*. a). Time and frequency resolution is 0.25 s and 3 Hz respectively.

The torsional resonance around 65 s (1950 rpm) - where the 10^{th} order hits the first natural frequency - and the horizontal line of the second natural frequency are really noticeable. Unfortunately, a 300 Hz disturbing signal (coming from the three-phase power inverter of the load motor) was also present during the measurement, but the rate of the first natural frequency is determined well by the resonant peaks around 320 Hz.



a) b)

Fig. 7. a) The Campbell plot of the torque signal during a 0 to 2400 rpm run-up, b) The amplitude density spectra of the torque signal during a 0 to 2400 rpm run-up

In order to determine the natural frequencies in a more appropriate way, the amplitude density spectra of the torque signal is presented in *Fig. 7*. b). Resonant peaks are dominant in the spectra allowing the convenient reading of the exact values of natural frequencies.

5. Conclusion

In this paper a novel method for experimental torsional modal analysis of shaft assemblies of PMSMs is presented.

In the first part of the article the existing torsional vibration measurement and excitation techniques are summarized, and a short review on the modeling of PMSMs is given. In the next chapter the applicability of a PMSM as a torsional exciter is investigated, and it is shown that in case of single-phase broadband current excitation a PMSM is a possible torque source for modal analysis purposes.

Before presentation of experimental modal analysis results, a tricky placement of the measuring accelerometer is proposed, that allows the angular acceleration to be easily measured in case of small angular displacement of the object.

Through the example of the first mode, the vibroacoustic effect of the cogging torque that can be modeled as a torsional spring with a position-dependent stiffness is treated.

Finally, the results of a torque signature analysis are presented. The good correlation between the natural frequencies of the shaft assembly confirms the applicability of the proposed torsional modal analysis method.

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