



# Dekonvolúció

3D Mikroszkópia és Dekonvolúció  
Iterációs eljárások

### Before and After Nearest Neighbor Deconvolution Analysis

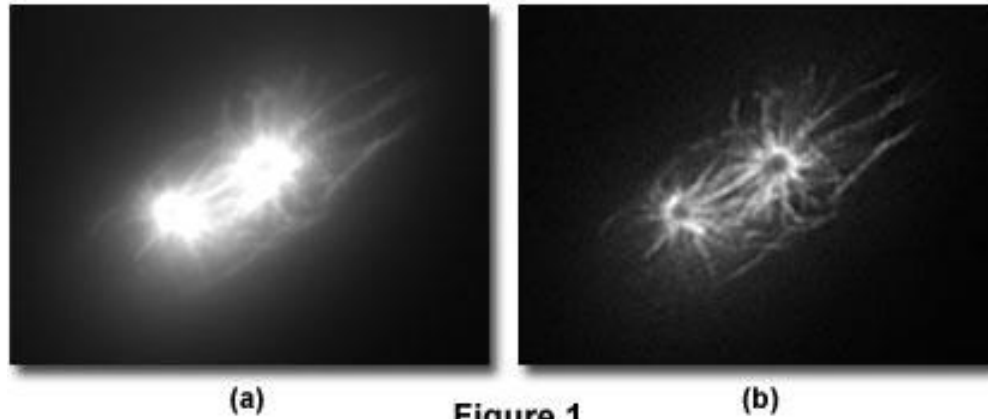


Figure 1

### Deblurring and Restoration Techniques

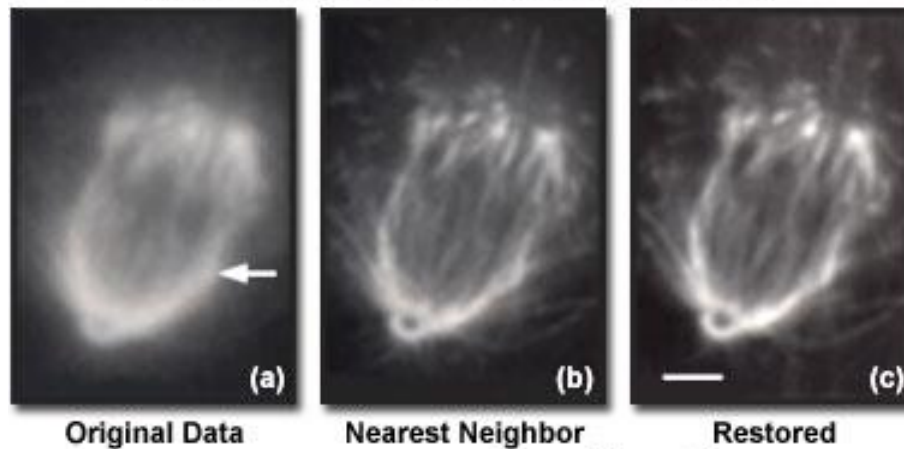


Figure 2

3D image restoration or reconstruction using a Fast Nearest Neighbor deconvolution algorithm. The parameter dialog box requires the source image file name, the point spread function (PSF) image file name. Because this algorithm uses information from the nearest image layers, it deconvolves much faster than the Maximum Entropy method.

### Deconvolution Algorithm Comparison

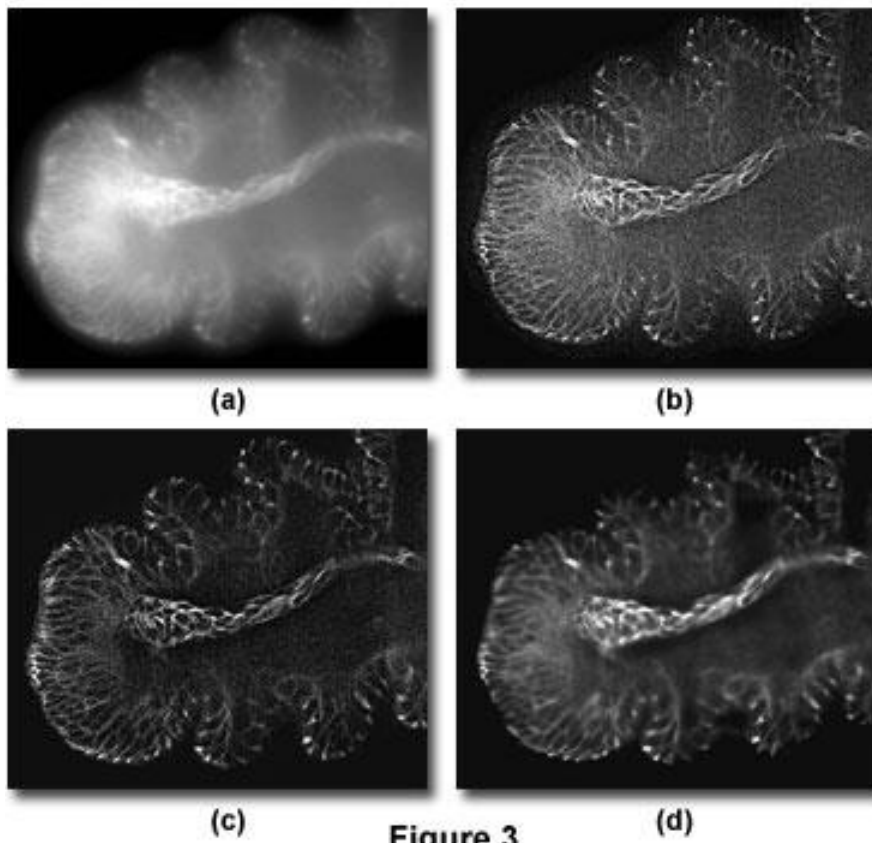
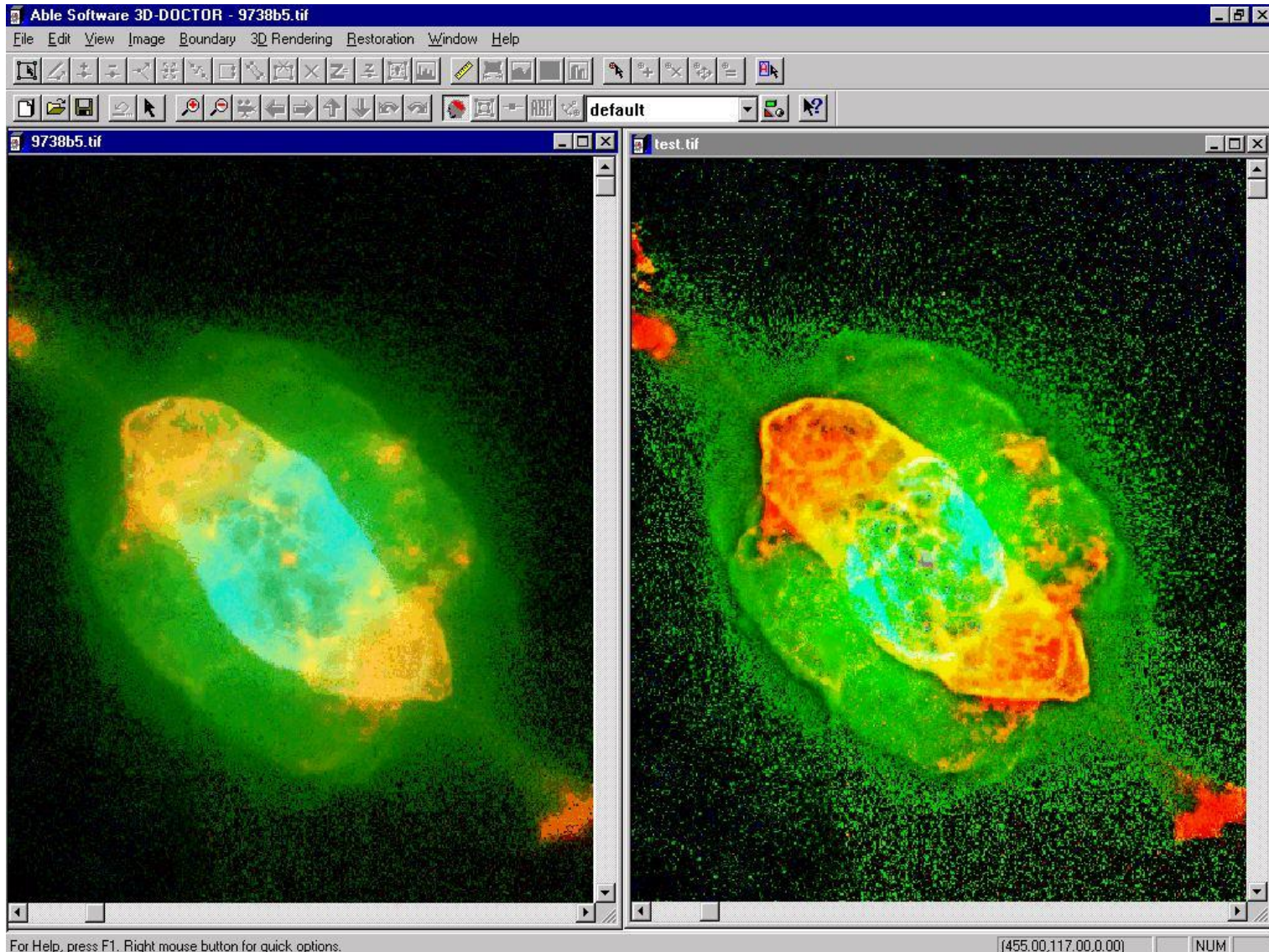


Figure 3

3D-DOCTOR: 3D image deconvolution is used to remove or reduce degradations that were incurred while the image was being obtained. These include the blurring introduced by optical systems and by image motion, as well as noise due to electronic and photometric sources: <http://www.ablesw.com/3d-doctor/recon.html>



$$\mathbf{W} = \mathbf{H} * \mathbf{V} + \mathbf{L}$$

$$\mathbf{R} = \mathbf{W} - \mathbf{H} * \hat{\mathbf{V}} - \mathbf{L} = \mathbf{W} - \mathbf{H} * \mathbf{X} - \mathbf{L}$$

$\mathbf{V}$ ,  $\mathbf{W}$  are the  $M \times N$  input and output images,  $\mathbf{H}$  is the convolution kernel, and  $\mathbf{L}$  is the bias. Since eq. is the canonical form of any linear shift-invariant filter, it covers imaging effects such as smoothing of the lens system, first-order optical errors, volume distortion of microscopic target, etc.

If the output of this linear filter is applied to the input of a restoration system, ( $\mathbf{W}=\mathbf{U}$ ), it transforms the input into the original unfiltered image in the least square error sense, i.e.  $\mathbf{X}_{t=\infty} = \hat{\mathbf{V}}$ .

This can be proved using the well-known gradient iteration method, considering square norm of  $\mathbf{R}$ :

$$\mathbf{R} = \mathbf{U} - \mathbf{H} * \mathbf{X} - \mathbf{L}$$

$$\mathbf{G}_X = \frac{d\|\mathbf{R}\|}{dX} = 2\mathbf{H}^r * (\mathbf{H} * X - \mathbf{U} + \mathbf{L})$$

Általában a minimalizálás regularizációs eljárássá egészül ki:

Hibatag: Becslési hiba +  $\lambda$  \* Kényszerfeltétel

- Élek aránya
- Magfüggvény maximalizálása
- Max. Eltérés az eredetitől

The mathematical formulations that we used in the image recovery problem is stated as follows [12]:

$$\min_u f(u) \equiv \min_u \frac{1}{2} \|h \star u - z\|_{L^2(\Omega)}^2 + \alpha \int_{\Omega} |\nabla u| dx dy \quad (1)$$

where  $\alpha$  is a parameter and  $\star$  denotes the convolution operator. It is known that TV regularization works effectively for recovering “blocky” images [6].

We formulate the blind deconvolution problem as

$$\min_{u, h} f(u, h) \equiv \min_{u, h} \frac{1}{2} \|h \star u - z\|_{L^2(\Omega)}^2 + \alpha_1 \int_{\Omega} |\nabla u| dx dy + \alpha_2 \int_{\Omega} |\nabla h| dx dy. \quad (2)$$

Here  $\alpha_1$  and  $\alpha_2$  are positive parameters which measure the trade off between a good fit and the regularity of the solutions  $u$  and  $h$ . Such an approach of using TV as a special case of anisotropic diffusion for recovering  $u$  and  $h$  is also employed

$$\frac{d\mathbf{X}}{dt} = \eta \mathbf{G}_{\mathbf{X}} = -\eta \mathbf{H}^r * \mathbf{H} * \mathbf{X}(t) + \eta \mathbf{H}^r * \mathbf{U} - \eta \mathbf{H}^r \mathbf{L}$$

$$\frac{d\mathbf{X}}{dt} \approx \eta \mathbf{G}_{\mathbf{X}} = -\eta \mathbf{H} * \mathbf{X}(t) + \eta \mathbf{U} - \eta \mathbf{L}$$

$$\mathbf{X}(t+1) \approx \mathbf{X}(t) - \mathbf{H} * \mathbf{X}(t) + \mathbf{U}, \quad \mathbf{L}=0, \eta=1, \mathbf{X}(t) = \mathbf{U}$$

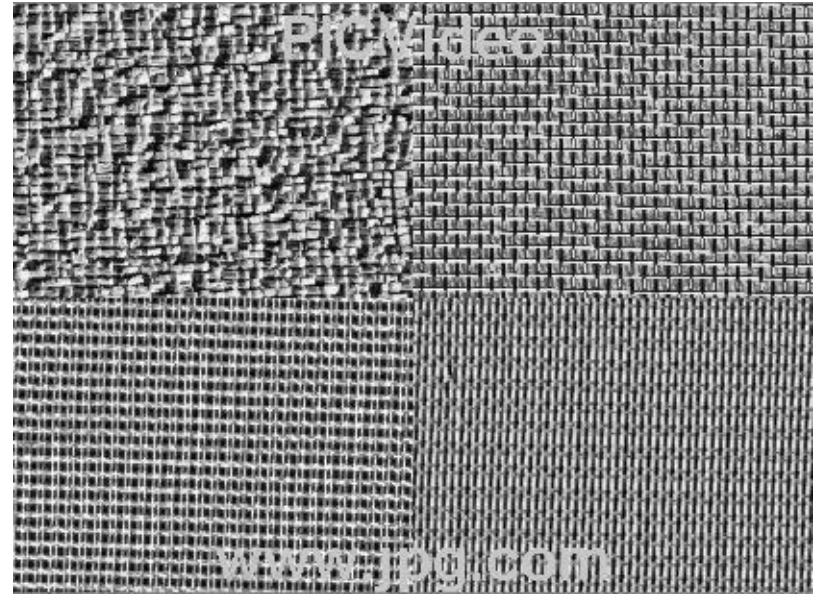
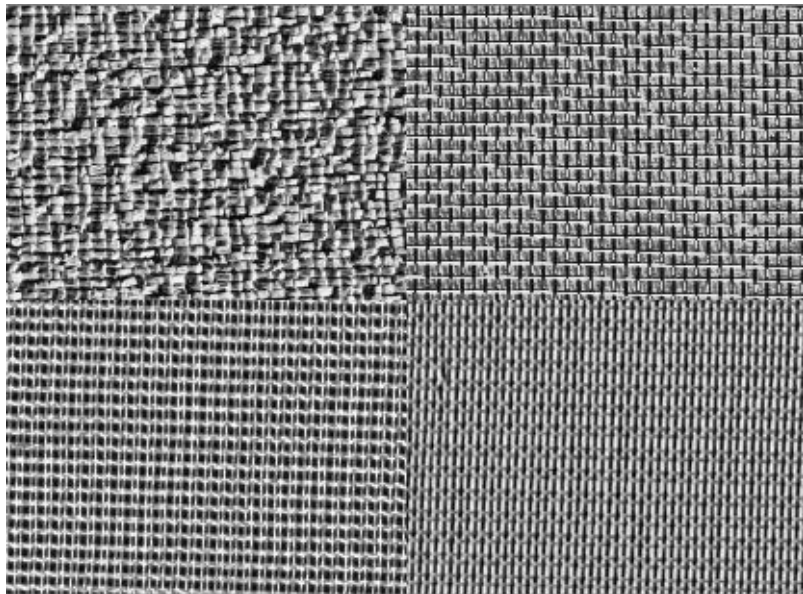
ÚjraBecsült = Becsült -  $\mathbf{H} * \text{Becsült} + \text{Mért}$

Here the  $\mathbf{H}^r$  convolution-matrix comes from a 180° rotation of the convolution-matrix  $\mathbf{H}$ . This gradient can be used in an infinite iteration process in calculating  $\mathbf{X}$  (IIR convolution filter). This time-dependent approximation process is given by the state-equation.

$\mathbf{H}^r * \mathbf{H}$  in the feedback is always stable if its system block-matrix ( $\#^T \#$ ) given by the  $\#$  hypermatrix of the  $\mathbf{H}$  convolution-matrix is nonsingular.



A visszacsatolt konvolúciót tartalmazó CNN **egyidejűleg** különböző műveleteket képes elvégezni, mint: konvolúció, dekonvolúció, mozgatás, korreláció és féltónusos kimenet, és ezzel különösen alkalmas képi textúrák analízisére.

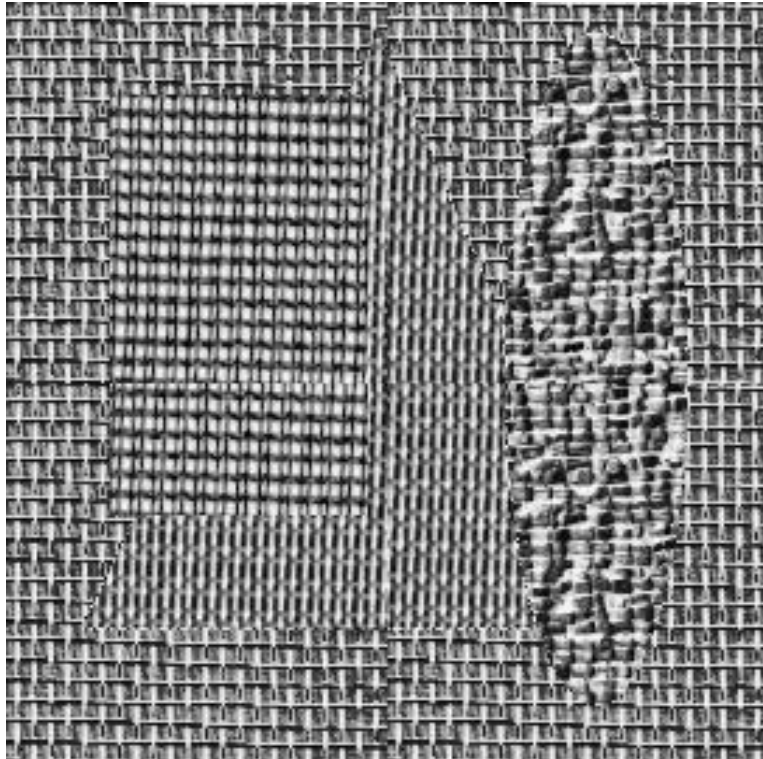


*Dinamikus szűrés,*  
genetikus algoritmussal  
előállított paraméterekkel

*4 bemeneti textúra*

# Textúra szegmentálás

bemeneti textúra

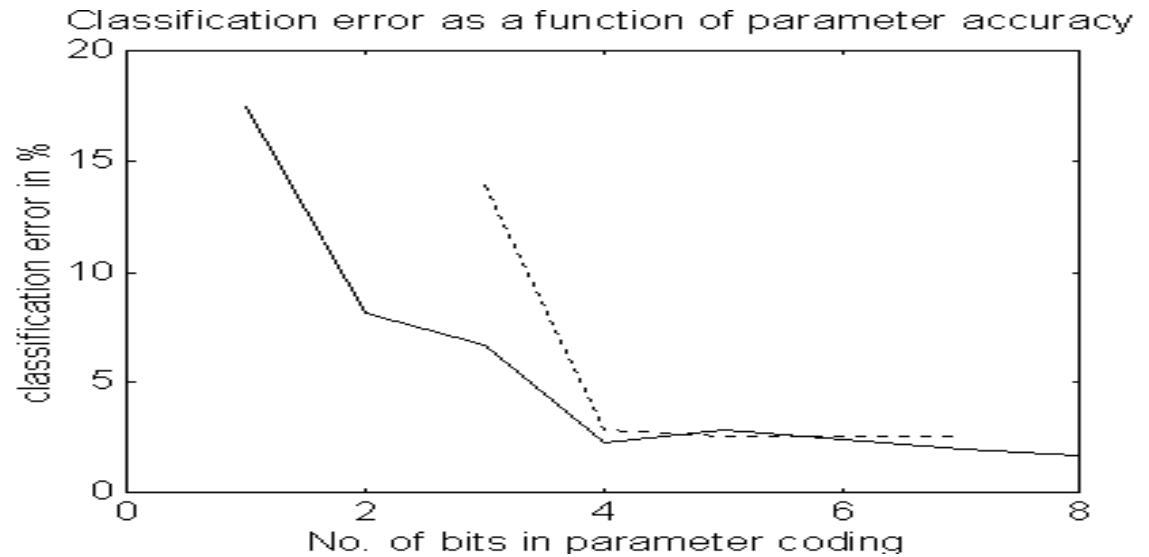


Szegmentált eredmény  
4 szűrés eredményeinek  
összevetésével

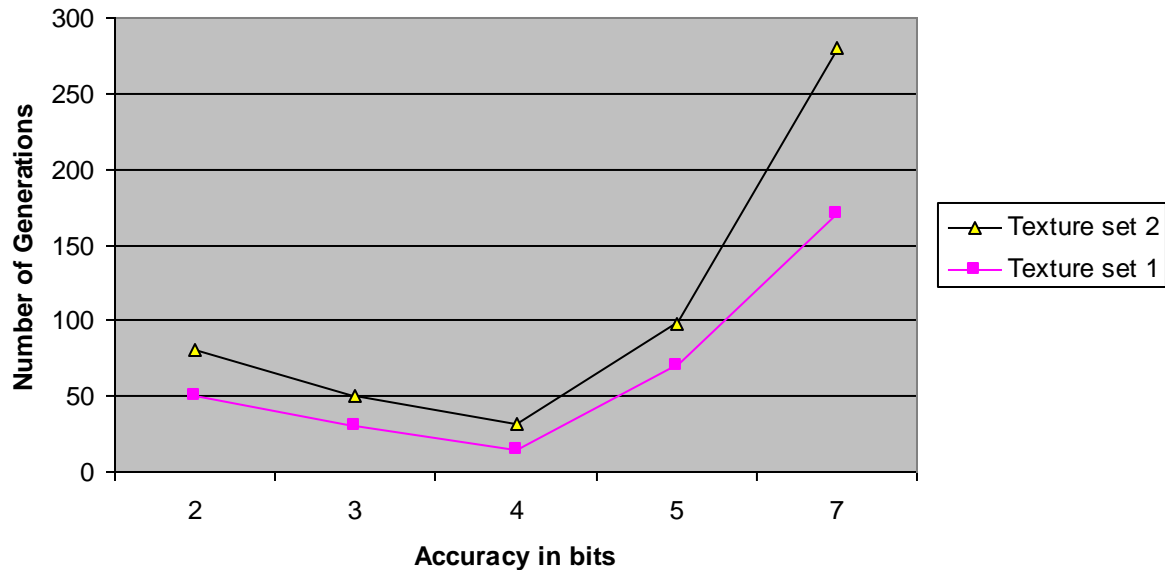


# Robusztusság:

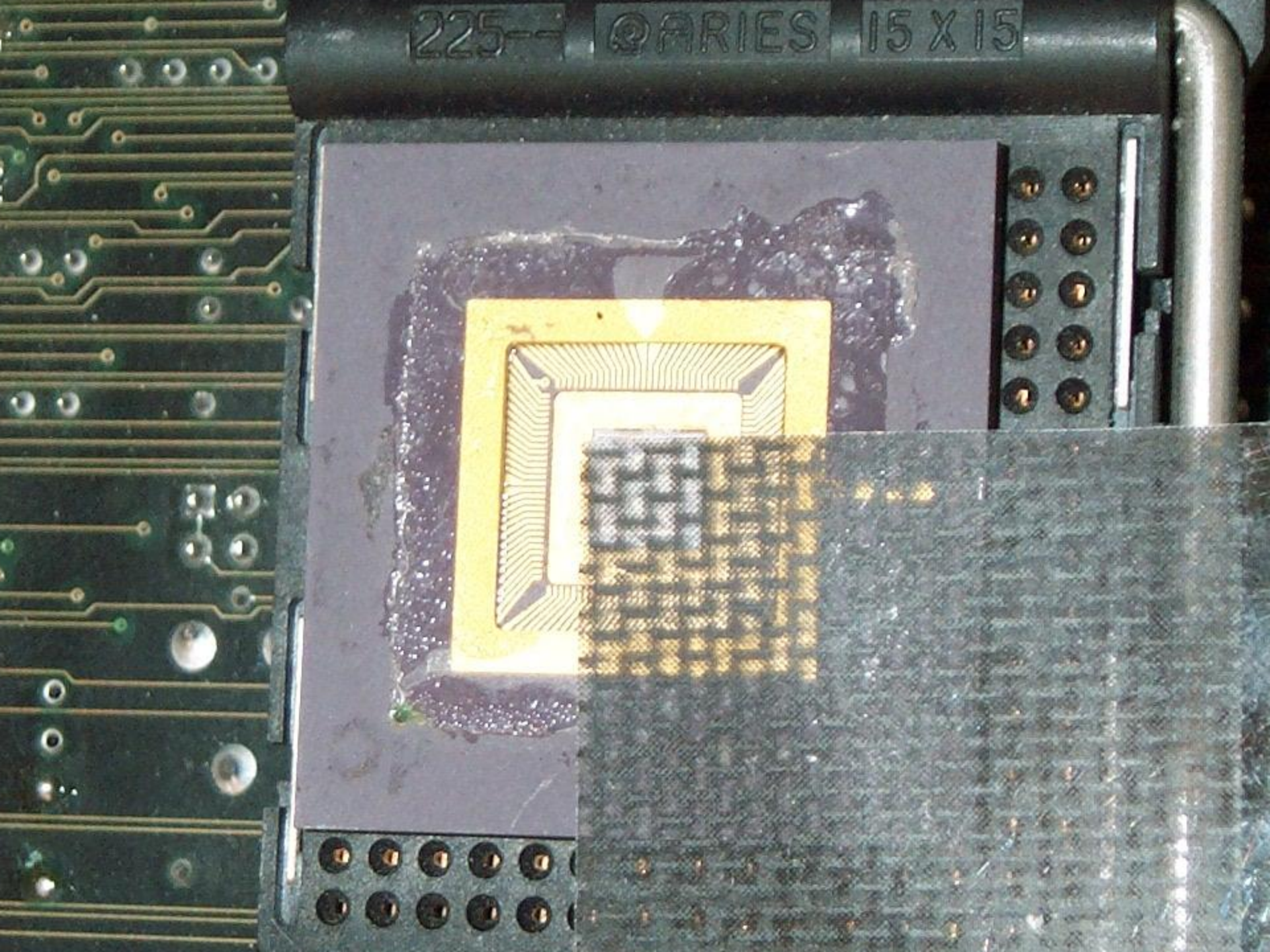
- Paraméterpontosság
- Optimalizálási idő



Effort of Genetic Algorithm



225-- @ARIES 15 X 15



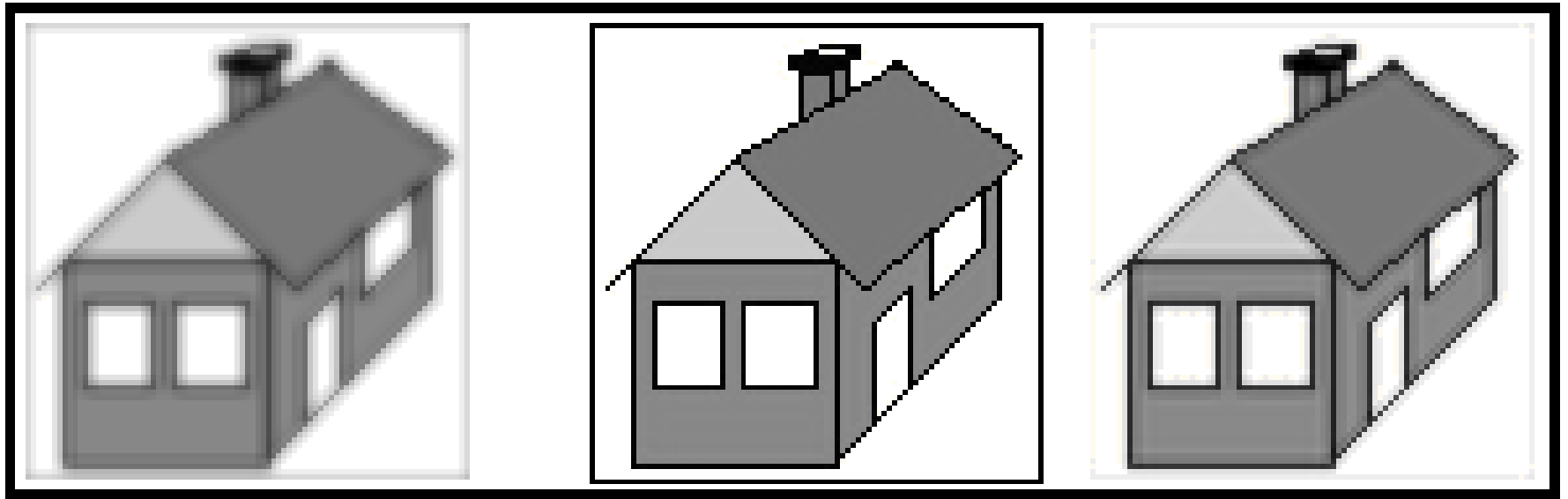
# Textúra felismerés a 64x64 CNN lapkán



A bemeneten **4**  
különböző  
mintázatot  
tapogatónk le



Szűrés után a bevett kép  
**átlagos** árnyalata a mintázat  
típusára lesz jellemző



Blurred image of a drawing with  $H$  of Table 1 (left);  
Reconstruction by  $H$  (mid) up to 30% variation in the  
parameters of  $H$  (right)

# Speciális esetek a stabil konvergenciára

$$\mathbf{H} = \mathbf{H}_1^r * \mathbf{H}_1$$

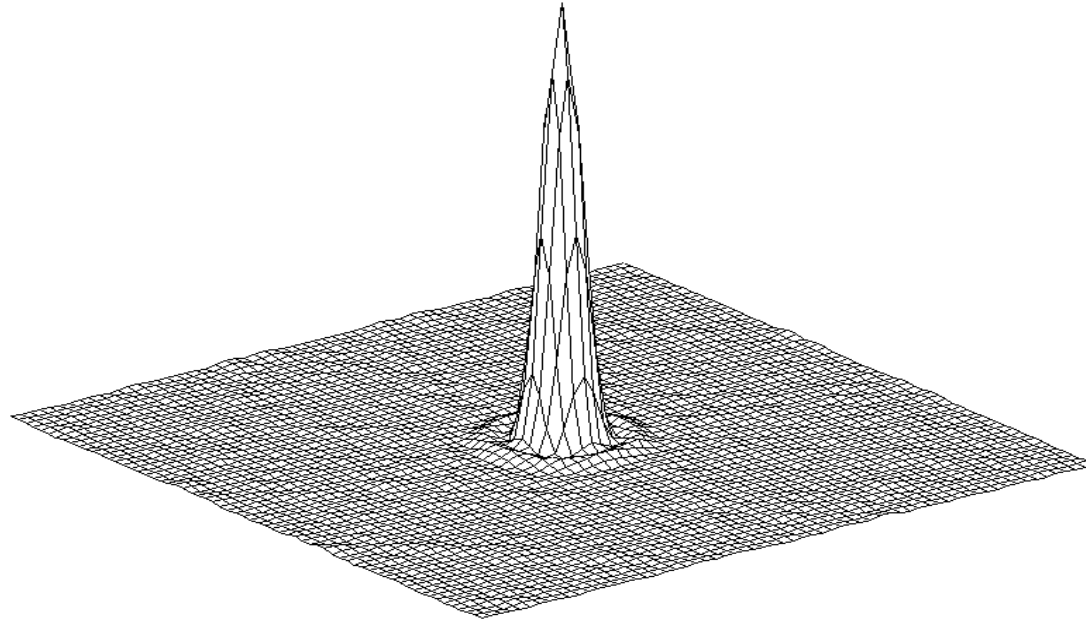
$$\mathcal{H} = \mathcal{H}_1^T \mathcal{H}_1$$

$$\mathbf{H} = \mathbf{H}_y * \mathbf{H}_x^T$$

$$\mathcal{H} = \mathcal{H}_x^T \otimes \mathcal{H}_y$$

In the stack matrix notation the elements of the separable PSF can be written into Kronecker direct product.

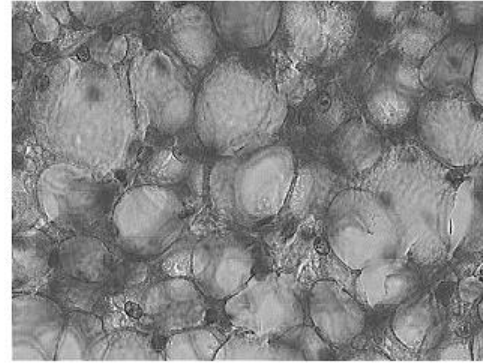
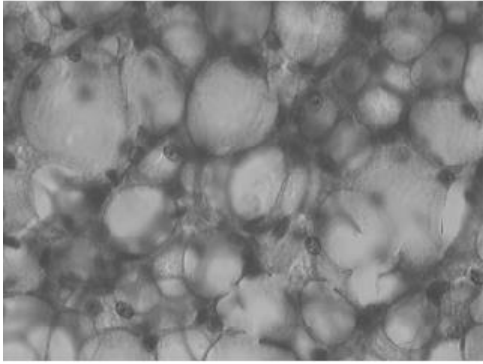
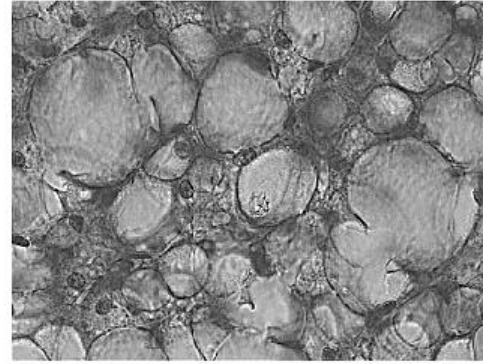
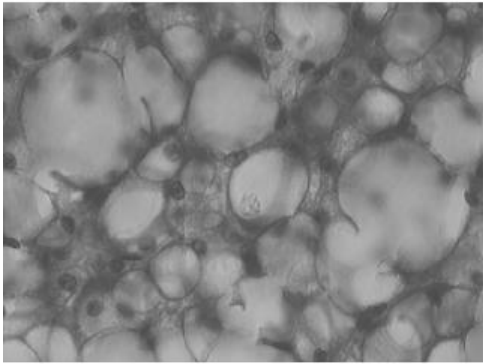
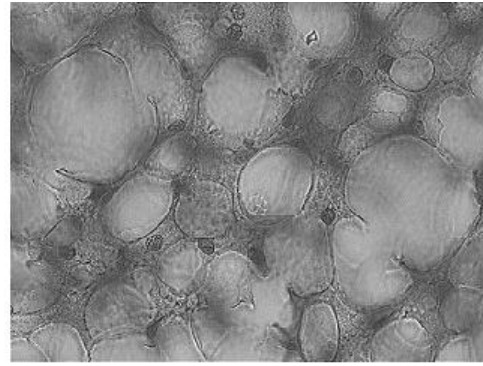
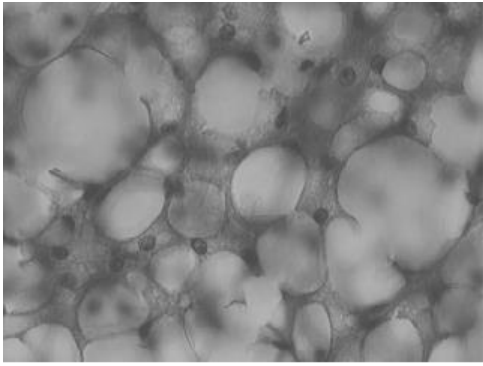
Since the eigenvalues of the Kronecker product are the products of the eigenvalues of the simple matrices, in case of positive definite matrices the resulting product term will also be positive definite. This implies a very weak constraint for the circular PSF.

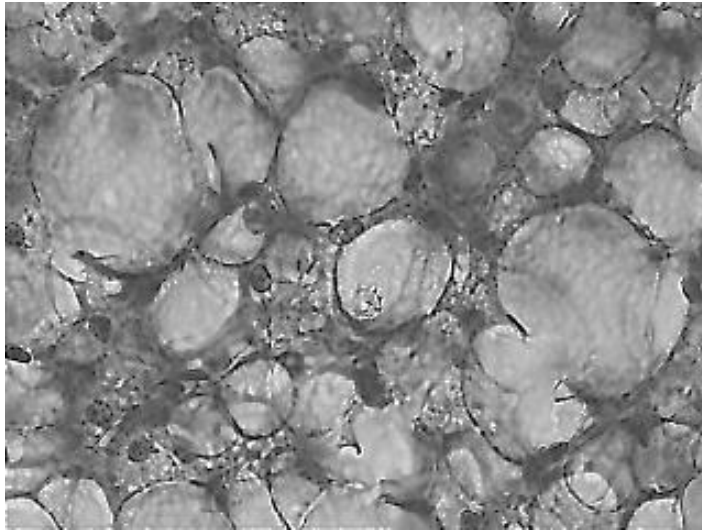


The 3D Point Spread Function of a defocused system as a function of the  $r$  and  $z$  coordinates

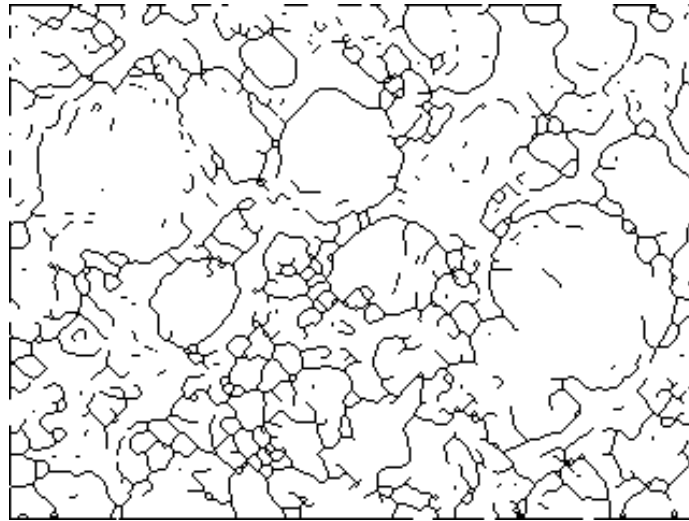
$$h(R, Z) = \mathfrak{F}_2\{P(\rho, z)\} = \mathfrak{F}_2\{e^{[i(2\pi/\lambda)\sigma(z)(\rho/\rho_0)^2]} \text{circ}(\rho/\rho_0)\}$$







Anisotropic Diffusion based edge enhancement and noise removal on the mid-layer



The skeletonized edge map of the microscopic mid-layer

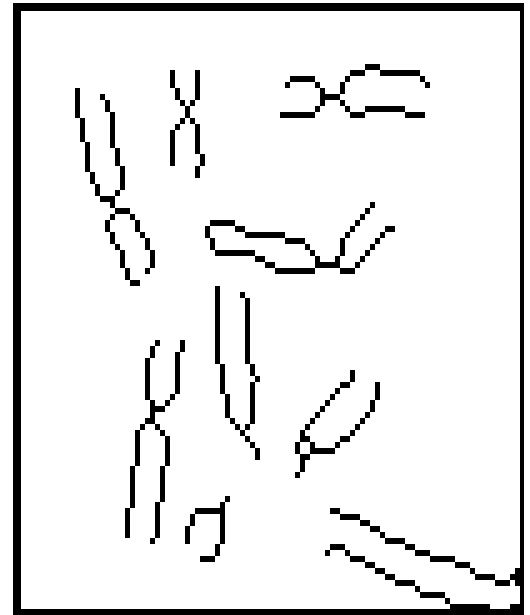
*Két- és három-dimenziós  
dekonvolúció visszacsatolt  
konvolúciós tér-szűrővel*



*Eredeti*



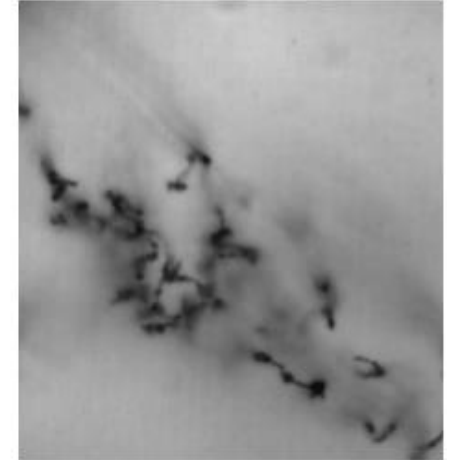
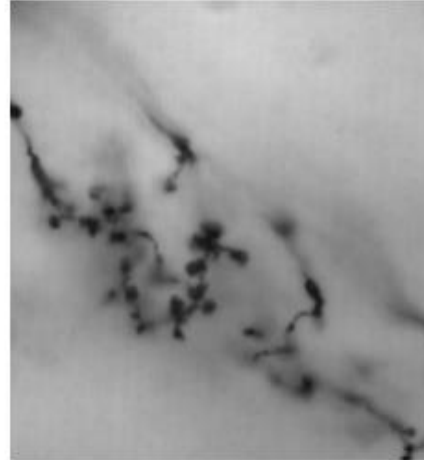
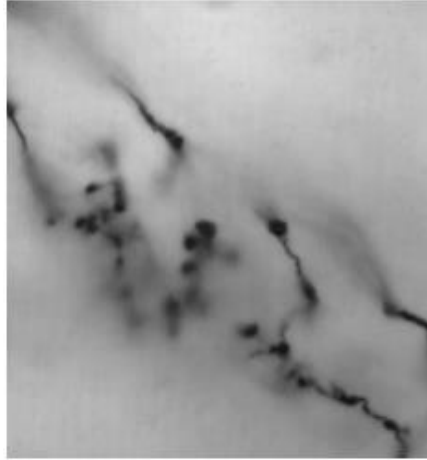
*Kiélesített*



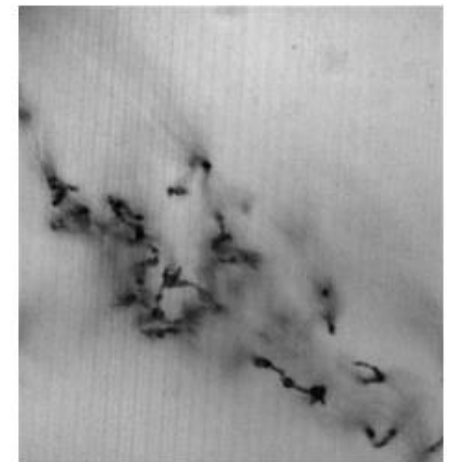
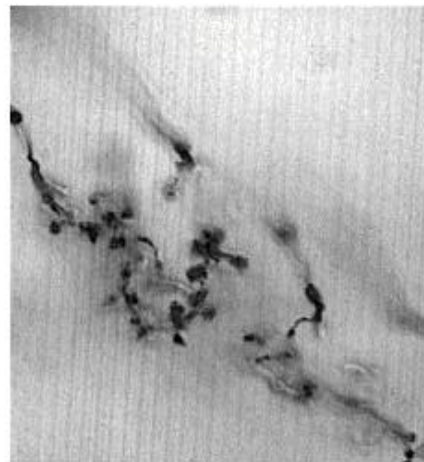
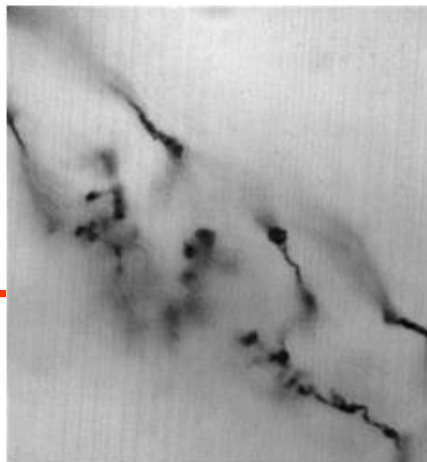
*Vázosított*

# Kép mélységi élesítése (Mikroszkópi felvételsorozat)

Látott  
kép-  
sorozat



Kiélesített  
számító-  
gépes  
eredménykép-  
sorozat



# Blind Deconvolution

## *Vak dekonvolúció*

- Valószínűségi Bayes modell
- Kapcsolat az energialeloszlás és az optikai átviteli függvény (PDF) között
- Kettős iteráció a
  - A konvolúciós hatás
  - A képEgyüttes becslésére
- Becslés a fókusztérképre

# Focus estimation with deconvolution

- Blind deconvolution:
  - Given observation  $g$ , give an estimation of the original image  $f$  and the blurring function (PSF)  $h$  :  $g = f * h$
  - Starting from Richardson's original formula based on Bayesians:

$$P(f_i|g) = [P(g|f)P(f_i)] / \sum_j [P(g|f_j)P(f_j)]$$

$$P(f|g) = P(fg)/P(g)$$

$$P(f_i) = \sum_l P(f_i g_l) = \sum_l P(f_i|g_l)P(g_l) = \sum_l \frac{P(g_l|f_i)P(f_i)P(g_l)}{\sum_j P(g_l|f_j)P(f_j)}$$

$$P_{k+1}(f_i) = P_k(f_i) \sum_l \frac{P(g_l|f_i)P(g_l)}{\sum_j P(g_l|f_j)P_k(f_j)}$$

# Blind deconvolution with Lucy - Richardson double iterations

$$f_{i,k+1} = f_{i,k} \sum_l \frac{h_{i,l} g_l}{\sum_j h_{j,l} f_{j,k}} = f_{i,k} \sum_l h_{i,l} \frac{g_l}{\sum_j f_{j,k} h_{j,l}}$$

$$f_{k+1} = f_k \left( h_k \otimes \frac{g}{f_k \otimes h_k} \right)$$



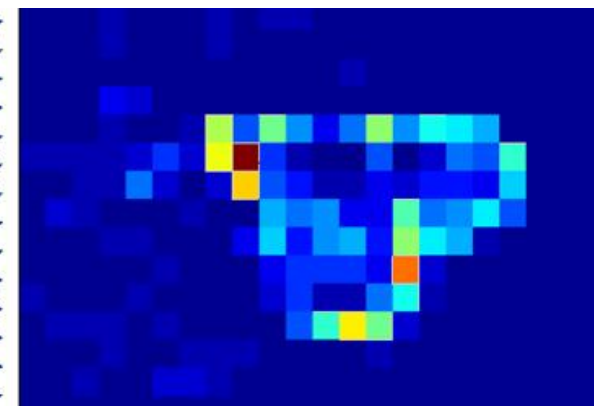
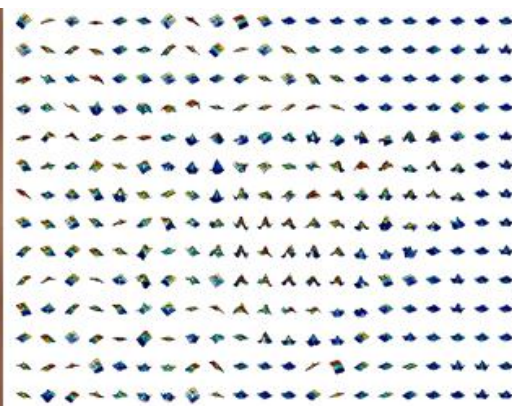
# The double iteration

- We create a localized double iteration scheme for locally varying  $f$  and PSF estimation ( $r$  – location vector):

$$\begin{cases} f_{k+1}(r) = f_k(r) \left[ h_k(r) * \frac{g}{g_k}(r) \right] \\ h_{k+1}(r) = \frac{h_k(r)}{\gamma} \left[ f_k(r) * \frac{g}{g_k}(r) \right] \end{cases}$$



- $f$  and the PSF vary locally according to the amount of blur (distortion) present on the image locally
- Stop the double iteration at a finite step (here #5) and check the **error between the measured and the estimated blurred image** blocks:  $\|g - g_k\|$
- Is MSE usable for comparison the BD residual errors of different blocks?



# Constraints and ill-posedness

- In the local deconvolution we consider only a few constraints
  - symmetricity,
  - non-negativity,
  - zero phase.
  - and **nothing** about the image content **regularization** (e.g. edges). Localized deconvolution runs on small blocks, range of the PSF. Thus the ill-posed iteration process tends to be **noisy**.
- For the classification we stop at a low iteration count and we need a stable error measure which gives different values for differently focused areas, and which is not much affected by the process's noisy nature.

ADE : angle deviation error

Orthogonality criterion: signal and noise are independent

$$\left| \arcsin \frac{\langle \mathbf{g}, \mathbf{g} - \mathbf{g}_k \rangle}{|\mathbf{g}| \cdot |\mathbf{g} - \mathbf{g}_k|} \right|$$

In case of

$$\mathbf{g} - \mathbf{g}_k = [ +1, -1, -1, +1, -1, +1 \dots -1, +1 ]$$

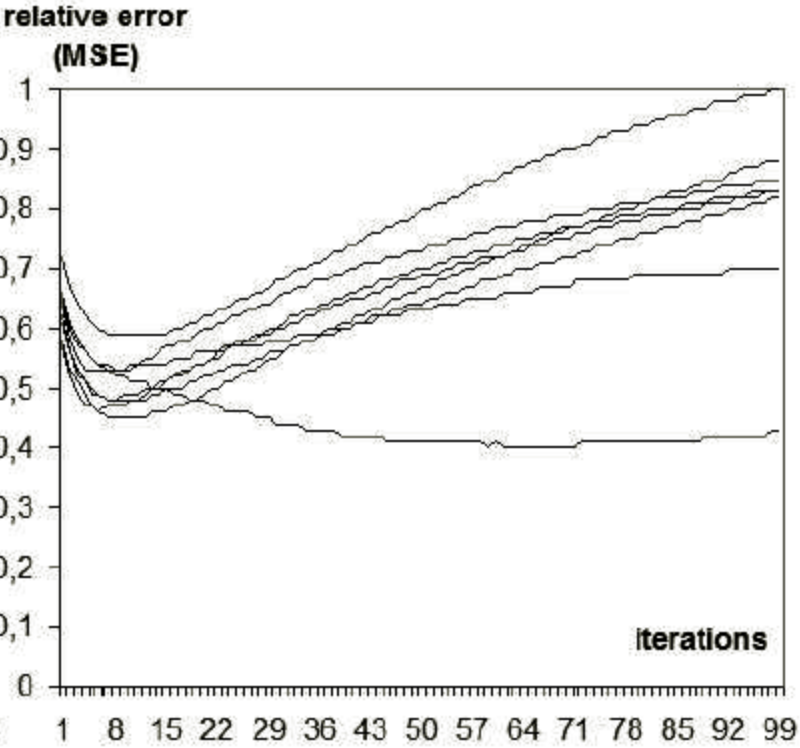
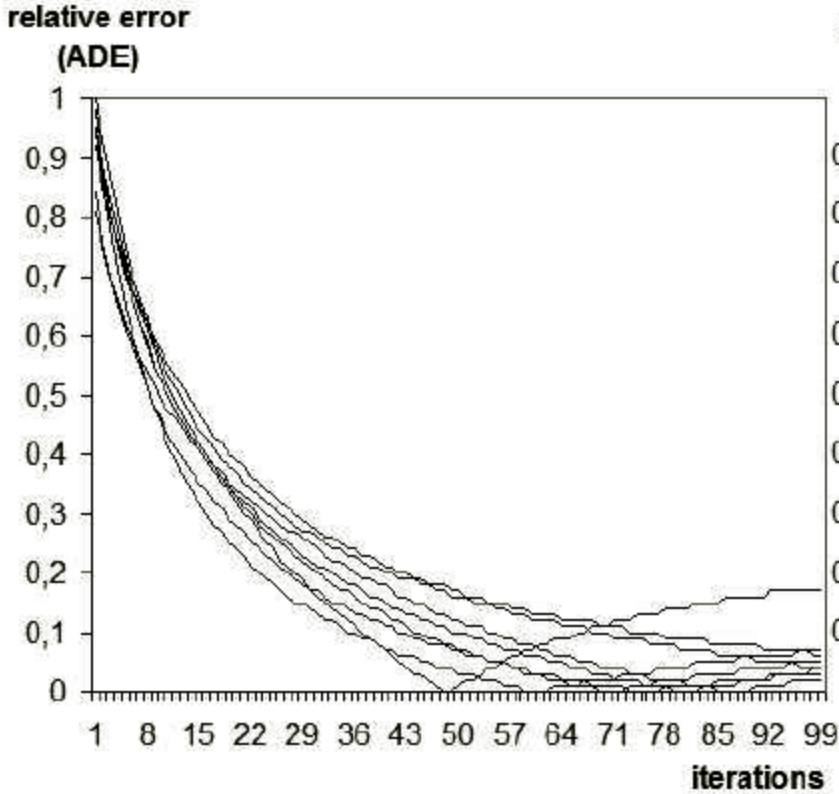
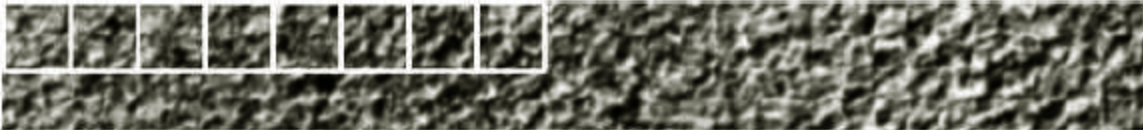
$$\mathbf{g} = [10, 10, 10, 10, 10, 10 \dots 10, 10]$$

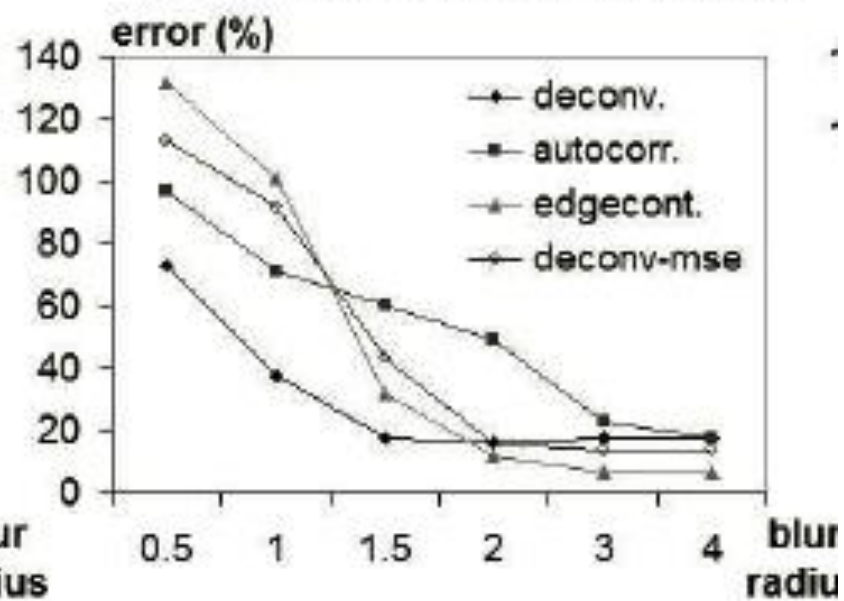
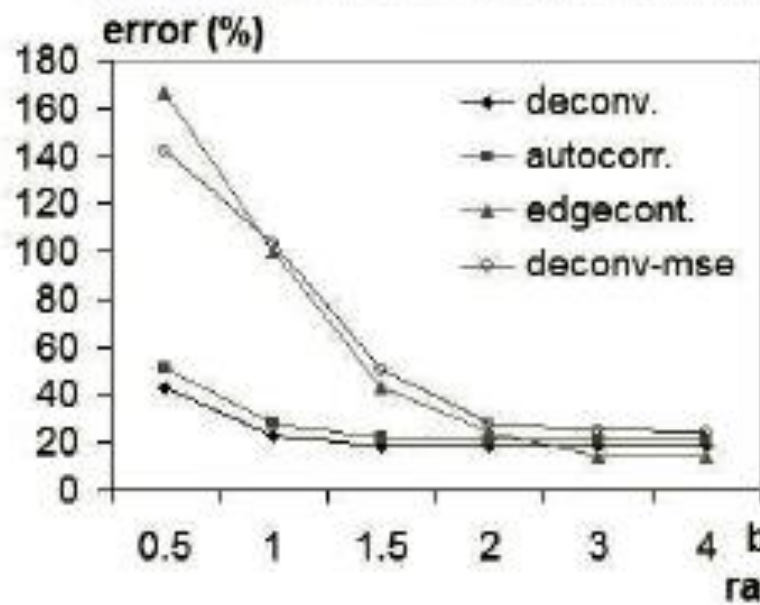
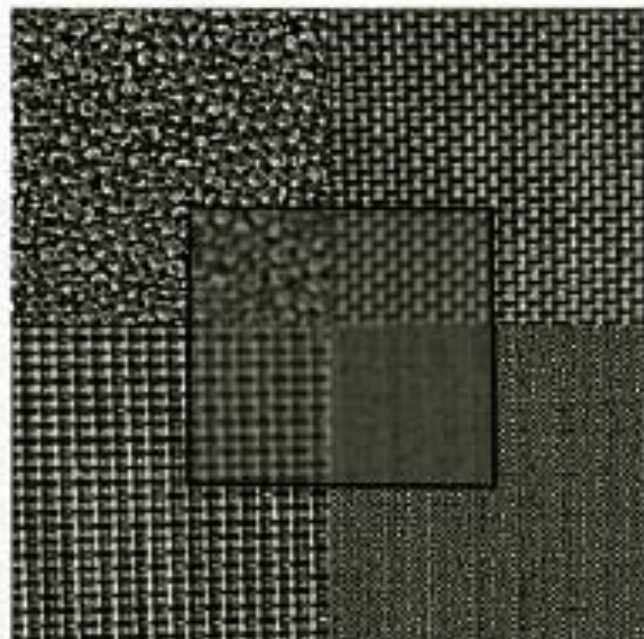
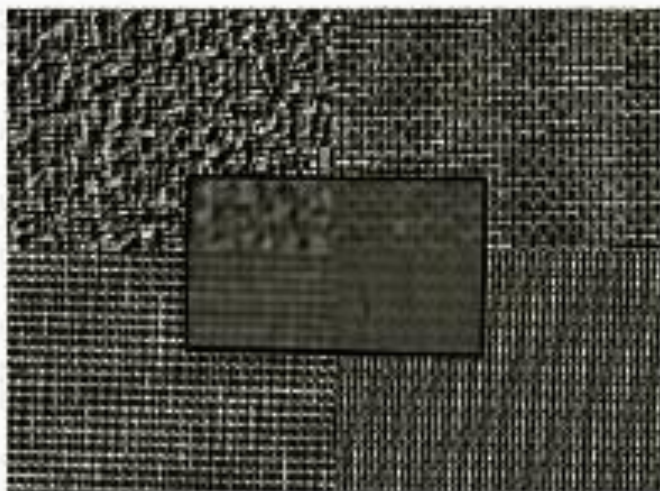
$\| \mathbf{g} - \mathbf{g}_k \|$  is high, while

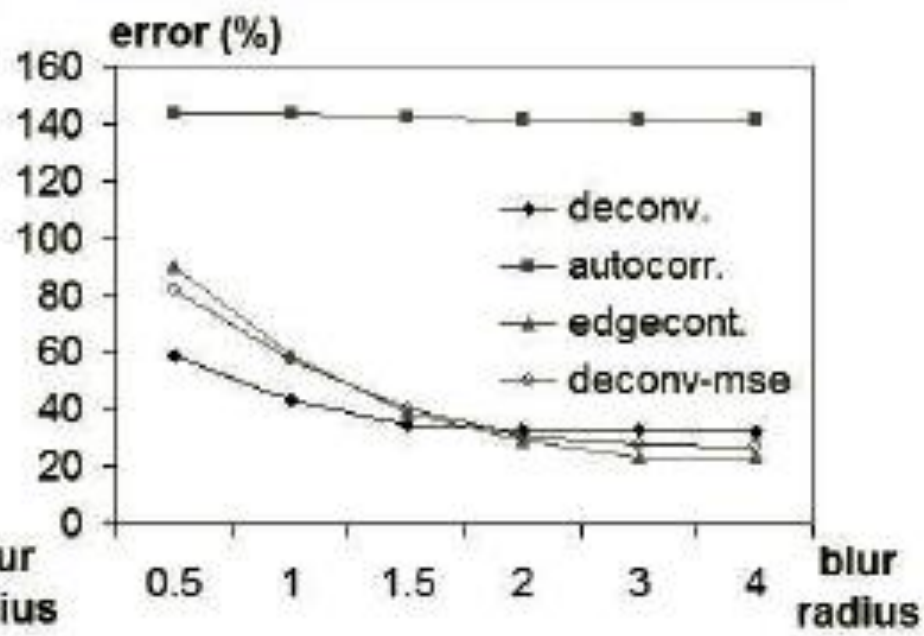
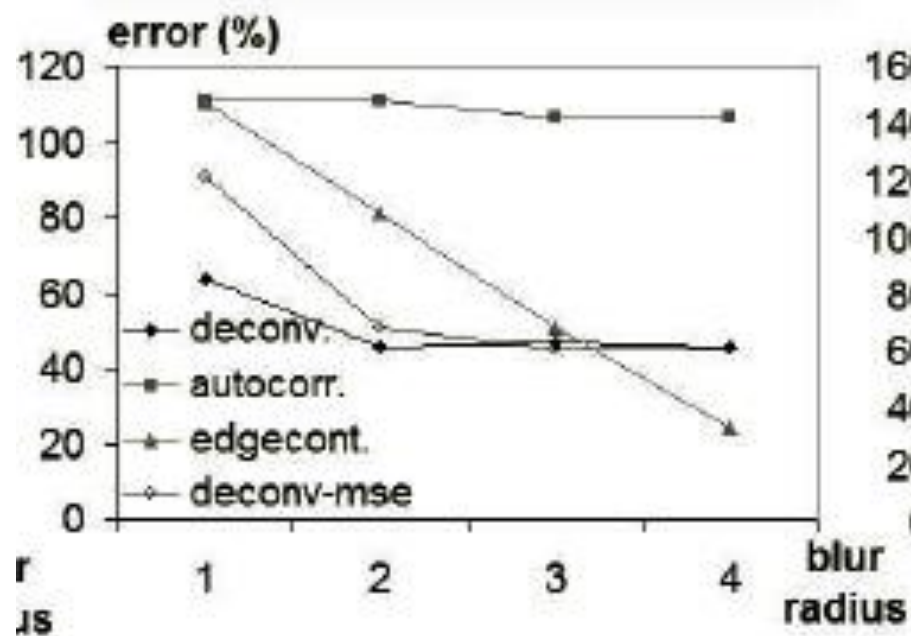
$$\langle \mathbf{g}, \mathbf{g} - \mathbf{g}_k \rangle \rightarrow \text{zero}$$

**Error curves for 8 neighboring blocks (each curve stands for one block) on a blurred texture sample (top) for the same blur with ADE (left), and MSE (right) .**

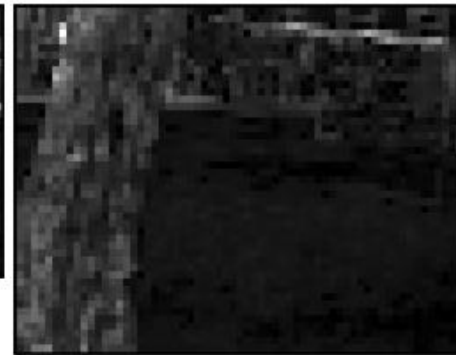
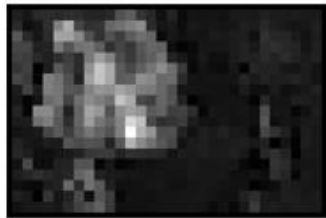
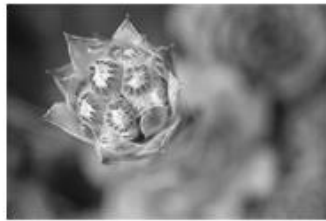
**Ideally, curves of the same measure should remain close to each other.**







Find images with similar relative focused objects:



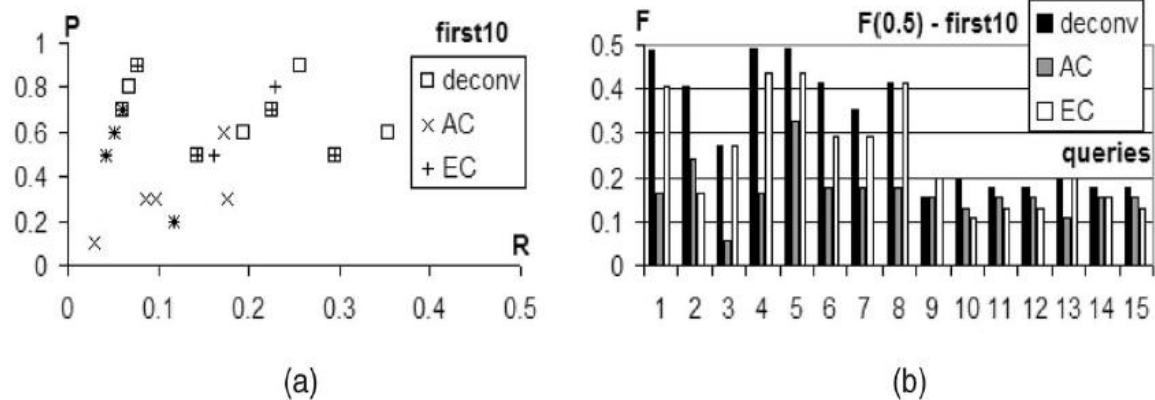


Fig. 14. Precision-Recall graph for 15 queries: (a) The returned closest 10 matches and (b)  $F(0.5)$  measure values (higher is better).

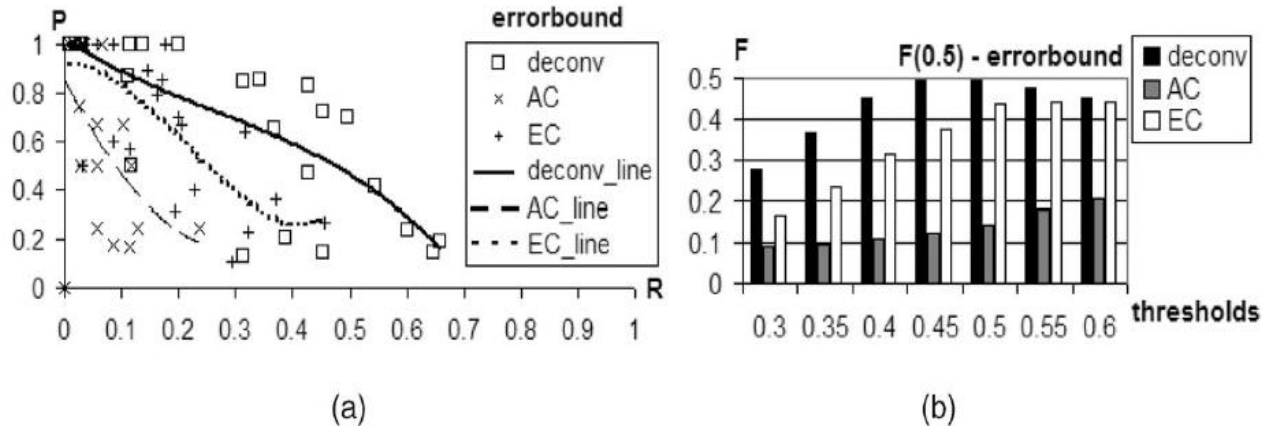


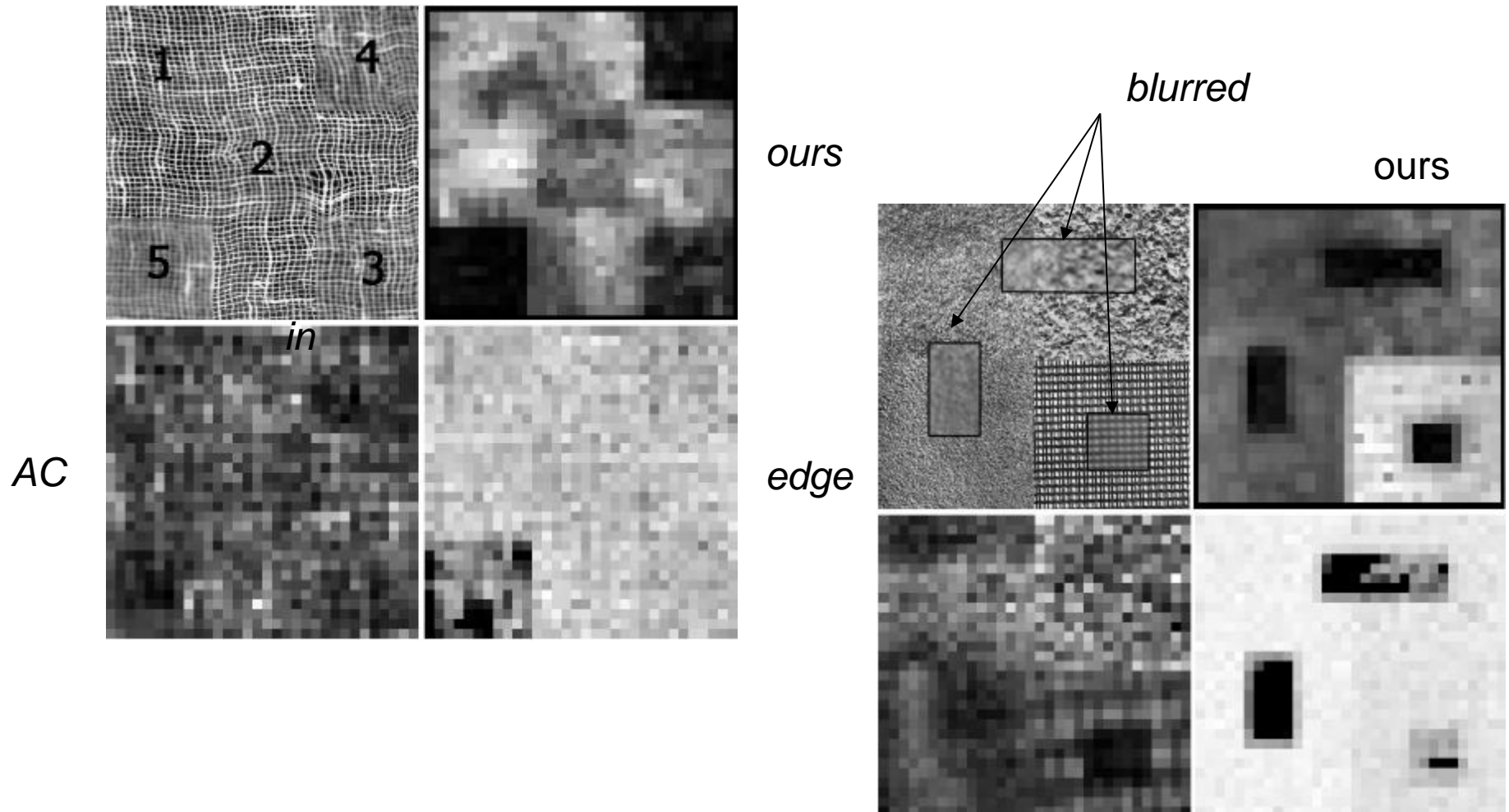
Fig. 15. (a) Averaged Precision-Recall curves for returning every match above a threshold (*errorbound* approach) for the 15 query images and (b) the respective  $F(0.5)$  measure values for the different thresholds (higher is better).



- Relative focus maps

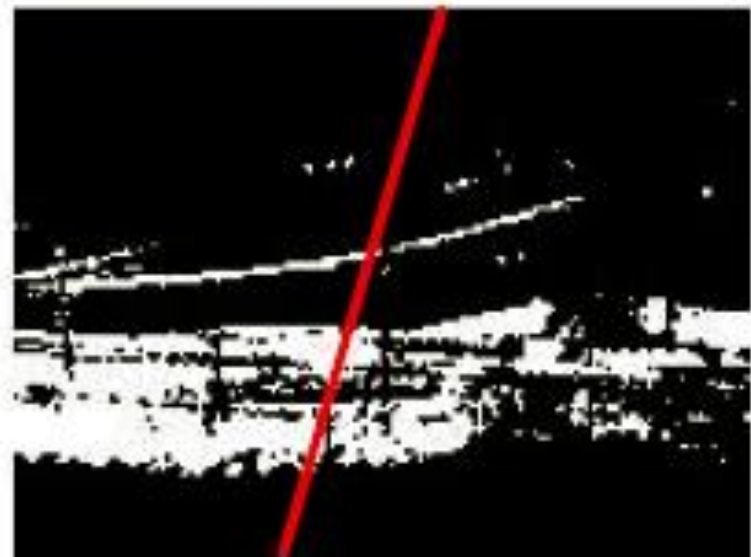
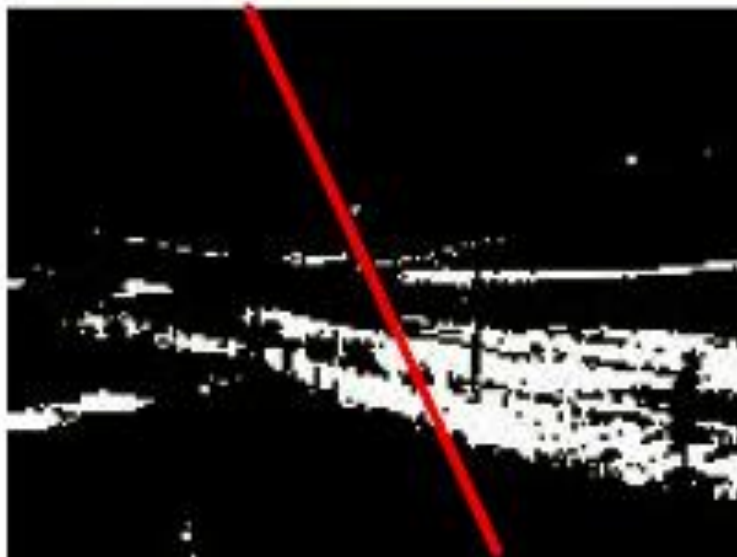


- Rel. focus map extraction - comparison on textures





Samples from input videos and  
Results of the *entropy* based preselection of feature points



Probabilistic interaction among points of different views  
for motion / no-motion functions

$$P(m_{1i} | m_{2k}) = \frac{1}{\sum_{t=1}^T b_{2k}(t)} \sum_{t=1}^T b_{1i}(t) b_{2k}(t)$$

$$P(m_{1i} | m_{2k}) = \frac{P(m_{2k} | m_{1i}) P(m_{1i})}{\sum_j P(m_{2k} | m_{1j}) P(m_{1j})}$$

Ergodic regular Markov chain has a unique stationary distribution

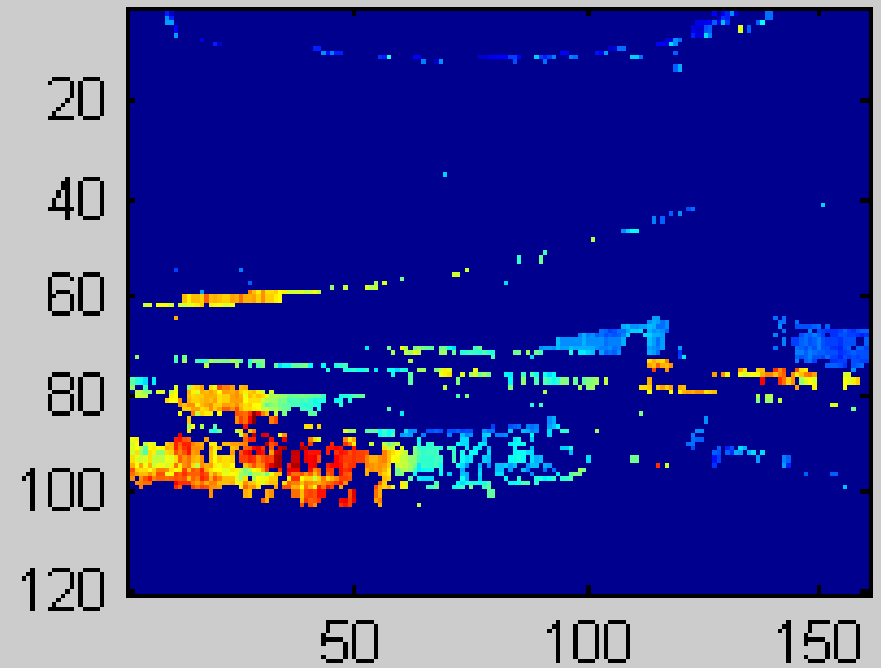
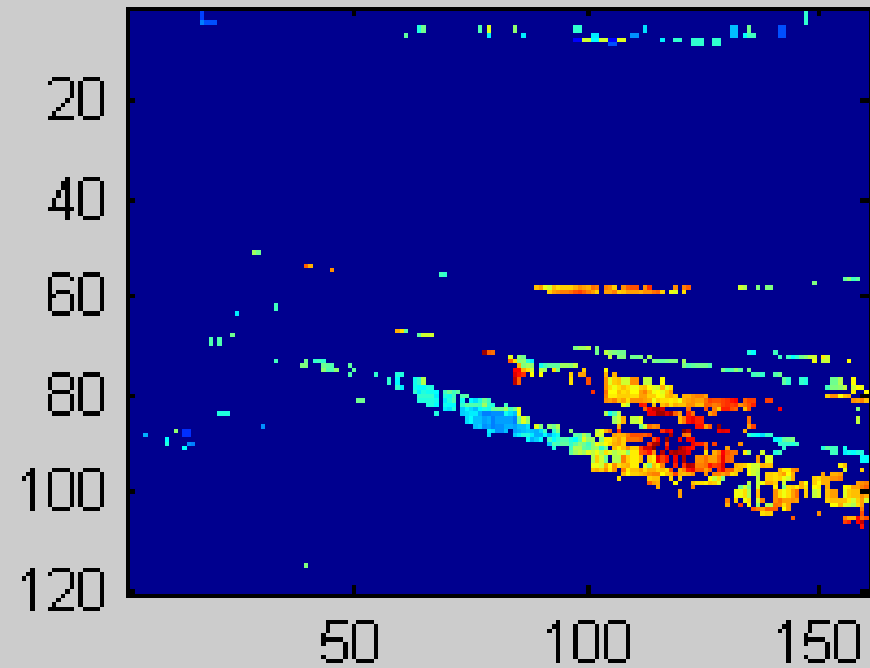
$$\begin{pmatrix} \underline{p}_1 & \underline{p}_2 \end{pmatrix} = \begin{pmatrix} \underline{p}_1 & \underline{p}_2 \end{pmatrix} \underline{\underline{\Pi}}$$

$$P(m_{1i})_{r+1} = P(m_{1i})_r \sum_k \frac{P(m_{2k} | m_{1i}) P(m_{2k})_r}{\sum_j P(m_{2k} | m_{1j}) P(m_{1j})_r}$$

$$P(m_{2k})_{r+1} = P(m_{2k})_r \sum_i \frac{P(m_{1i} | m_{2k}) P(m_{1i})_r}{\sum_j P(m_{1i} | m_{2j}) P(m_{2j})_r}$$

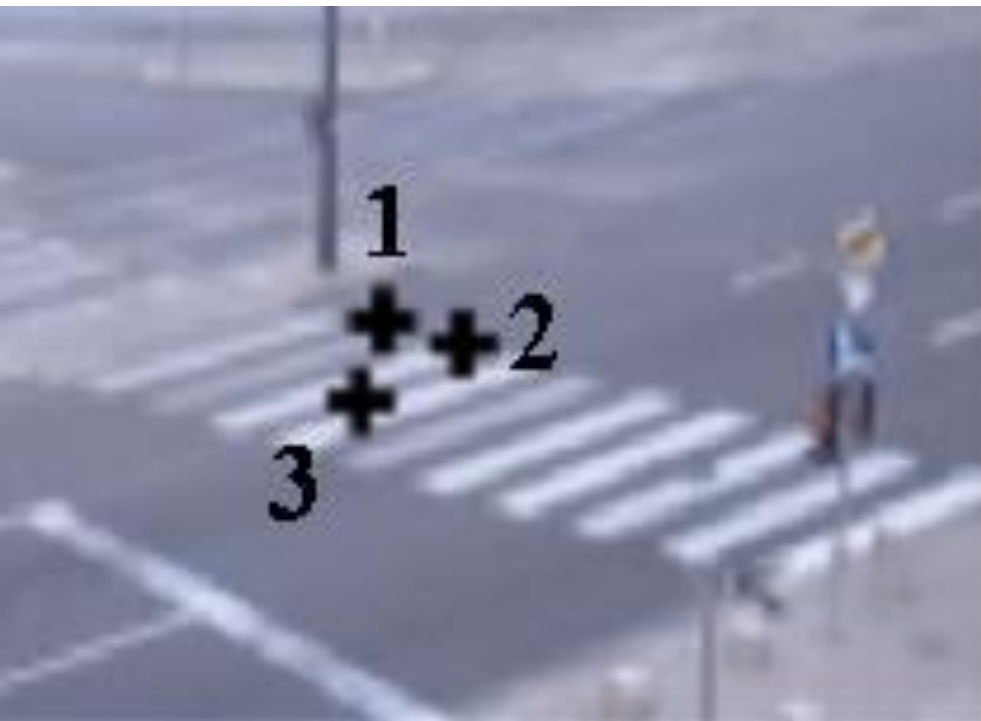
# Bayesian iterations of Ergodic regular Markov chain

with a unique stationary distribution



Sample point pairs obtained by Bayesian iterations.

The nearly corresponding points are numbered with the same number.







# The error function

- Localised blind deconvolution for focus map estimation:
  - run local deconvolution with a low iteration count
  - calculate local residual errors, with contrast weighting

$$E_r(g, g_k) = \text{arc sin} \frac{\langle g - g_k, g \rangle}{|g - g_k| \cdot |g|} \cdot \frac{C_r(g_r)}{\max_r \{C_r(g_r)\}}$$

$$C_r(g_r) = \frac{g_{\max\{x \in T_r\}} - g_{\min\{x \in T_r\}}}{g_{\max\{x \in T_r\}} + g_{\min\{x \in T_r\}}}$$

- use the local residuals for relative classification of areas

$$F(r) = \frac{c \cdot (E_r(g, g_k) - \min\{E_r(\cdot, \cdot)\})}{\max\{E_r(\cdot, \cdot)\} - \min\{E_r(\cdot, \cdot)\}}$$