Theoretical acoustics – proposed homework topics

Péter Rucz

2024

You can choose your homework from the following proposed topics. All the following homeworks involve performing numerical simulations using the acoustical Boundary Element Method. Each homework contains the following steps.

- Create a simple mesh using the program `gmsh`.[1]
- Perform Boundary Element simulations using the mesh and evaluate the acoustic pressure fields on the surface of the mesh and some field points. Use the reference BEM package (used during the course for demonstrations) to perform the computation.
- Evaluate the results by comparing to expectations / analytical results / approximations depending on the specific task.
- Write a short report on the results including your solution strategy, the results displayed in diagrams and a summary of the evaluation. You can also include your Matlab code.

1. **The cat's eye**

   The cat's eye geometry is a typical test geometry for validating boundary element implementations. The cat's eye is a sphere with one octant removed, thus, it has a smooth outer surface and sharp edges and an inner corner too, as shown in Figure 1.

   Your task is to simulate the radiation of the cat's eye with the outer spherical boundary vibrating with a constant normal velocity of $v_n = 1\,\mathrm{m/s}$. The detailed description of the cat's eye benchmark problem is found at the EAA website.[2] Compute the pressure field of the cat's eye at the field points given in the case description document, from $5\,\mathrm{Hz}$ to $500\,\mathrm{Hz}$ with a resolution of $\Delta f = 5\,\mathrm{Hz}$.

2. **Plane wave scattering from a rigid sphere**

   Scattering of a plane wave from a rigid sphere is an idealized example of acoustical scattering from obstacles. This particular example also has an analytical solution that is attained as a sum of spherical Bessel functions. The scattered field is found as the infinite sum

$$p_{\mathrm{sca}}(r, \theta) = \sum_{l=0}^{\infty} A_l h_l^{(2)}(kr) P_l(\cos\theta), \tag{1}$$

   where the amplitude coefficients A_l are given as

$$A_l = -(2l+1)\mathrm{j}^l \frac{l j_{l-1}^{(1)}(kR) - (l+1) j_{l+1}^{(1)}(kR)}{l h_{l-1}^{(2)}(kR) - (l+1) h_{l+1}^{(2)}(kR)}. \tag{2}$$

[1] You can download `gmsh` from: `https://gmsh.info/bin/Windows/gmsh-4.12.2-Windows64.zip` (When using `gmsh`, be sure to export your mesh using the `Export` menu and choosing `Version 2 ASCII` format for the `.msh` mesh file. Version 2 `msh` files are handled by the current Matlab code.)

[2] `https://eaa-bench.mec.tuwien.ac.at/fileadmin/t/eaa/LA_Case2_CatEye.pdf`

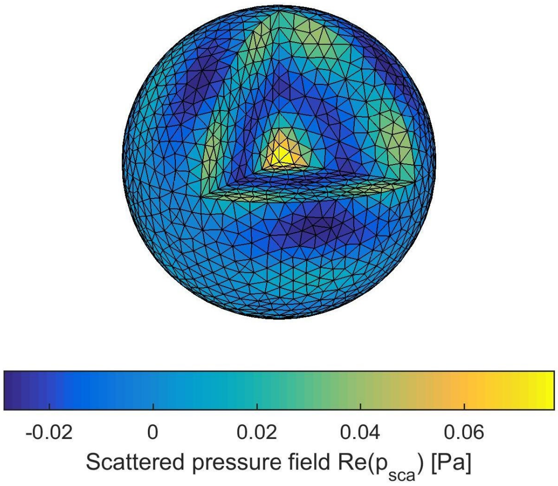

Figure 1: Scattered field of a point source from the cat's eye

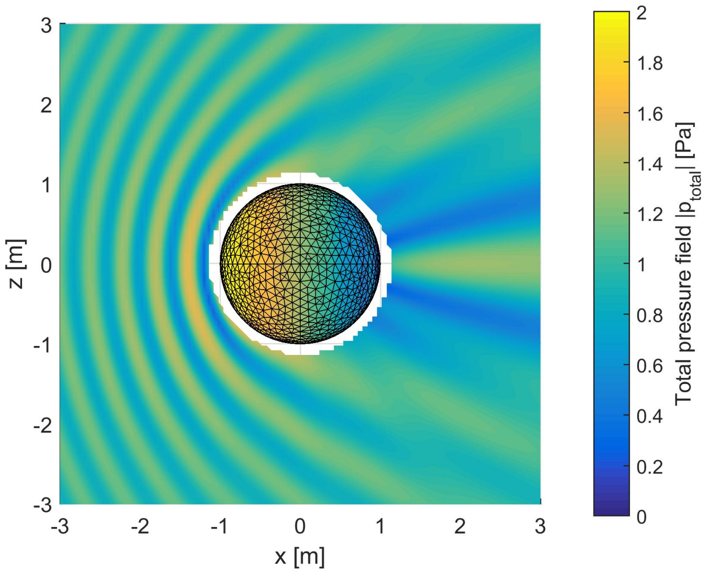

Figure 2: Scattering of a plane wave by a rigid sphere

Here, j are spherical Bessel functions, h are Hankel functions and P are Legendre polynomials. The sphere is centered at the origin and has radius R. The coordinate system is given by the spherical coordinates (r, θ, ϕ). The resulting field is cylindrically symmetric in ϕ.

The effect of the rigid sphere on the plane wave is illustrated in Figure 2. The task is to compute the scattered field of a plane wave hitting a rigid sphere at different frequencies and compare the results to the analytical solution of the problem.[3]

3. Radiation from a line source

The line source is an idealized model of an elongated source. The source is modeled as a

[3]Some more details on the problem can be found at `http://ansol.us/Products/Coustyx/Validation/MultiDomain/Scattering/PlaneWave/HardSphere/Downloads/dataset_description.pdf`

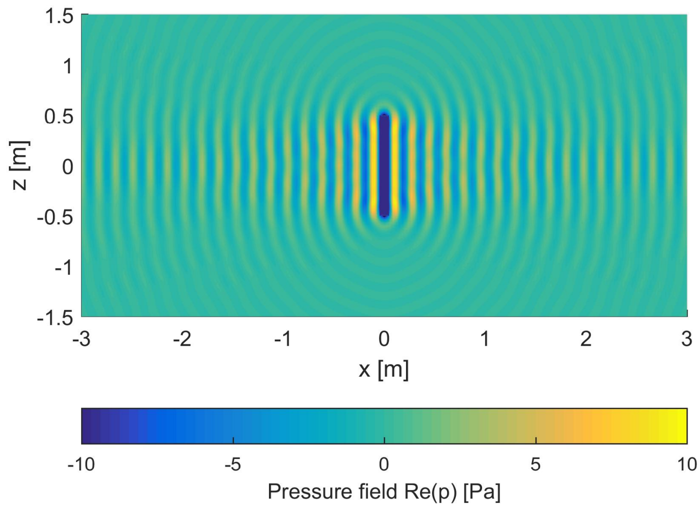

Figure 3: The radiated field of a line source model

circular cylinder that has a small radius and a vibrating lateral area. In the simplest case, the side area vibrates with a constant velocity. In this case, the far field directivity $D(\theta)$ of the finite line source is given as

$$D(\theta) = \frac{\sin\left(k\frac{L}{2}\sin\theta\right)}{k\frac{L}{2}\sin\theta}, \tag{3}$$

with L denoting the length of the line source. The angle θ is zero perpendicular to the axis of the line source. The near field radiation of a line source is illustrated in Figure 3.

Your task is simulate the radiation of a line source. Compare the directivities to the analytical formula in the far field and the near field at different frequencies.

4. **The loudspeaker revised**

   Loudspeaker boxes often contain more than one loudspeakers. Electrical filters are used to filter the input signal to drive the speakers in different ranges of frequency. In the higher frequency ranges membranes with smaller diameters are used.

   The radiation from a loudspeaker with one membrane at higher frequency is illustrated in Figure 4. Your task is to create a loudspeaker mesh with two membranes. Compare the directivities when the smaller, the larger, or both membranes are driven. Perform the comparison at different frequencies.

5. **Helmholtz resonator**

   The Helmholtz resonator is the simplest acoustical resonator consisting of a cavity of arbitrary shape and a neck with a small cross section. The natural resonance frequency of the Helmholtz resonator f_{hr} is determined as

$$f_{\mathrm{hr}} = \frac{c}{2\pi}\sqrt{\frac{S}{VL_{\mathrm{eff}}}}, \tag{4}$$

   where c is the speed of sound, S is the cross section area of the neck, V is the volume of the cavity, and L_{eff} is the effective length of the neck that includes the so-called end corrections due to radiation into open space.

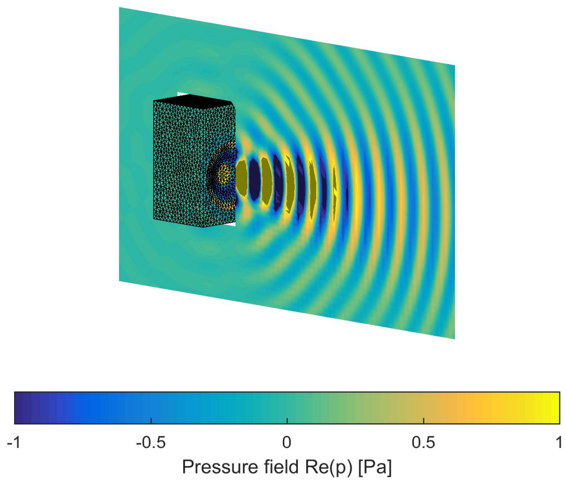

Figure 4: Radiation pattern of a loudspeaker at higher frequency

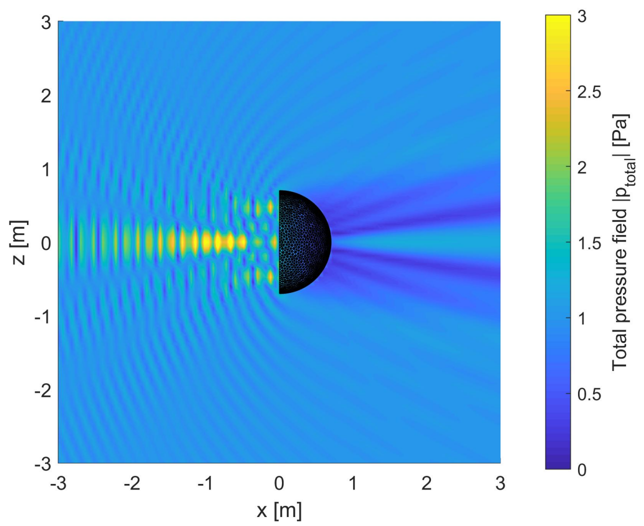

Figure 5: Scattering of a planar wave from a hemispherical reflector

Your task is to create the mesh of a Helmholtz resonator with the shape of your preference. Then, you can simulate the response of the Helmholtz resonator due to an external monopole sound source at different frequencies. Create a plot of the magnitude of the sound pressure at a point inside the cavity versus the excitation frequency. Compare the result of the BEM calculation with the analytical formula (4).

6. **Retroreflectors**

A retroreflector in optics is a device that reflects light with the minimum amount of scattering. One can also create an acoustical retroreflector that reflects pressure waves.

Your task is to create the mesh of an acoustical reflector (corner, parabolic, or hemisphere reflector) and calculate its reflection properties using a planar wave excitation at different frequencies and angles of incidence. Figure 5 shows the scattering of a planar wave from a hemispherical reflector.

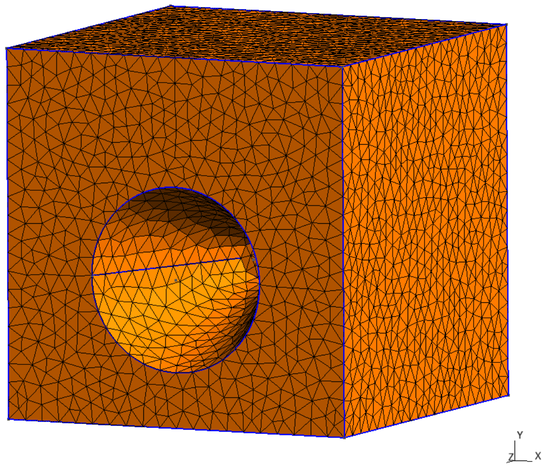

Figure 6: Example mesh of a loudspeaker with a conical horn

7. **Scattering from a prism**

   The task is to create the mesh of a wedge-shaped prism and evaluate the directivity of its scattered field at different frequencies and different angles of incidence.

8. **Loudspeaker with conical horn**

   In certain loudspeakers a horn is often is applied in order to enhance the efficiency of the acoustical radiation from the membrane.

   Your task is to create a mesh of loudspeaker built into a small enclosing box and investigate the effect of conical horns with different lengths and cone angles on the radiation characteristics of the speaker. Figure 6 shows an example horn configuration with cubic loudspeaker box.

9. **Own task**

   You can also simulate your own problem. In this case you should discuss the chosen problem with me.