

DWT/JPEG

Eadlőásvázlat

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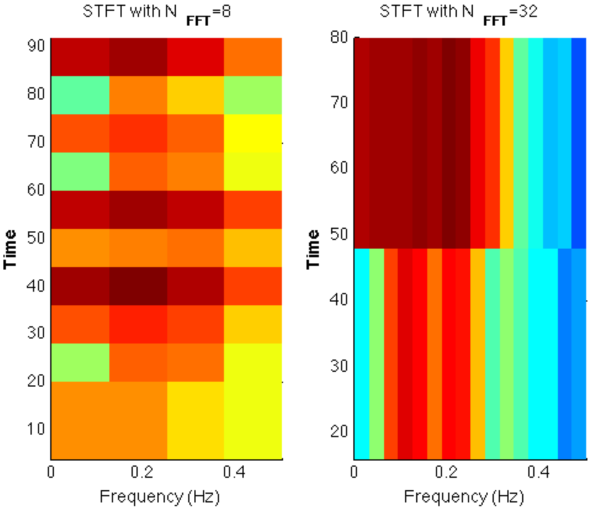
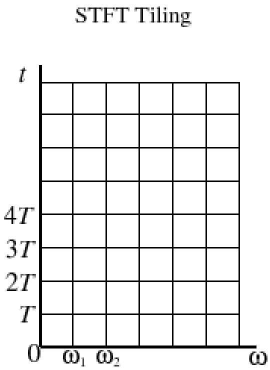
BME Hálózati Rendszerek és Szolgáltatások Tanszék

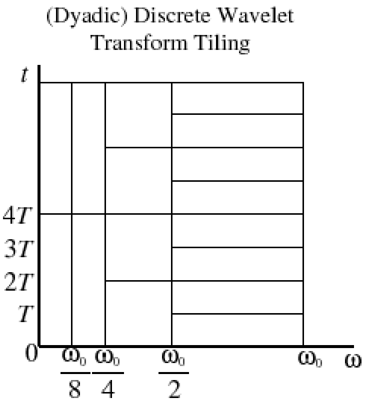
2015. szeptember

STFT


STFT with N FFT=8
90
80
70
60
50
40
30
20
10
Time
0
0.2
0.4
Frequency (Hz)
STFT with N FFT=32
80
70
60
50
40
30
20
Time
0
0.2
0.4
Frequency (Hz)

DFT vs DWT


STFT Tiling
t
4T
3T
2T
T
0
ω1
ω2
ω


(Dyadic) Discrete Wavelet
Transform Tiling
t
4T
3T
2T
T
0
ω0/8
ω0/4
ω0/2
ω0
ω

CWT

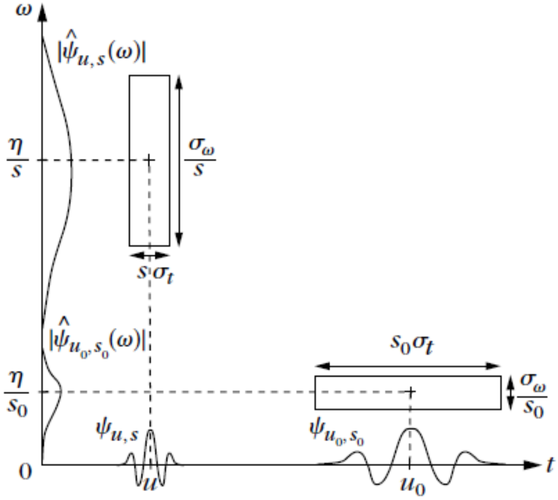
Egy folytonos $f(t)$, $-\infty \leq t \leq \infty$, jel folytonos Wavelet transzformáltja (CWT), melyet jelöljünk $W(s,\tau)$-vel, formailag a következőképpen definiálható:

$$W(s,\tau) = \int_{-\infty}^{\infty} f(t)\psi_{s,\tau}(t)\,dt,$$

ahol s a skála (scale), τ pedig az eltolás (shift). A $\psi_{s,\tau}(t)$ bázisfüggények az anya-waveletből származtathatók:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

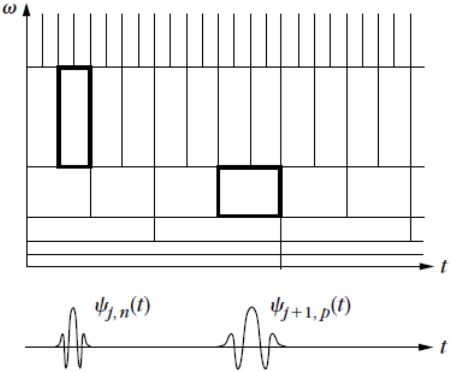
Wavelet ”multizoom”



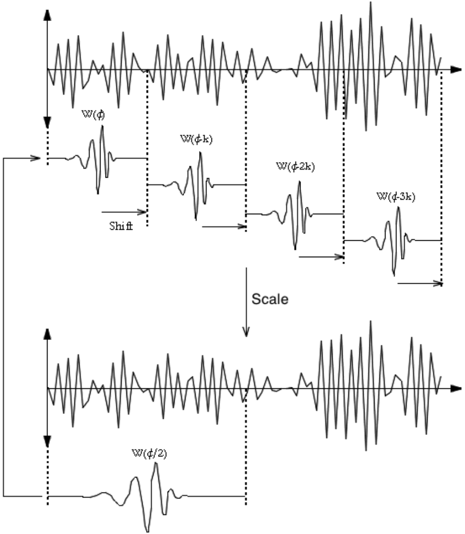
Függvények diadikus skálázása

$$\psi_{j,k}(t) = 2^{j/2}\psi(\frac{t-2^j k}{2^j})$$

$$\phi_{j,k}(t) = 2^{j/2}\phi(\frac{t-2^j k}{2^j})$$



Scale/shift értelmezése



DWT

$$DWT\{x[n]\} = W_\phi(j_0, k) + W_\psi(j, k),$$

ahol a skála (közelítő) együtthatók:

$$W_\phi(j_0, k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n]\phi_{j_0,k}[n],$$

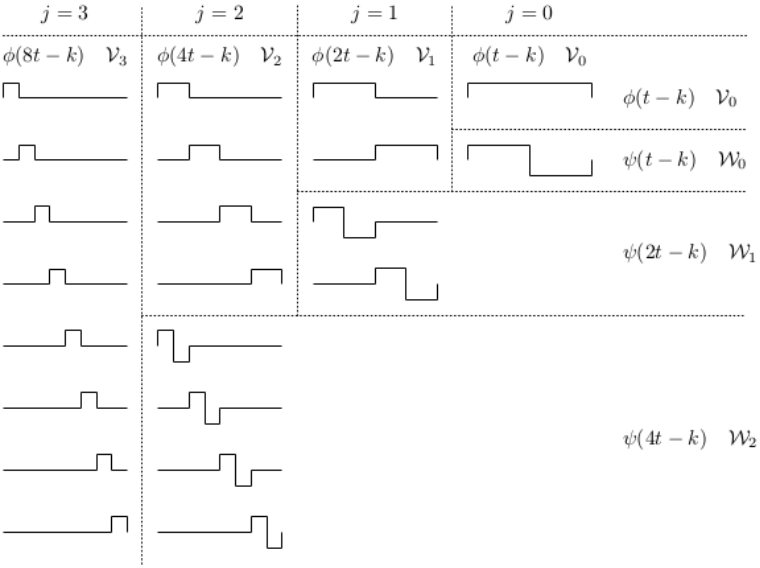
a wavelet (részlet) együtthatók pedig:

$$W_\psi(j, k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n]\psi_{j,k}[n] \qquad j \geq j_0.$$

Az inverz DWT (rekonstrukció) pedig a következő módon írható fel:

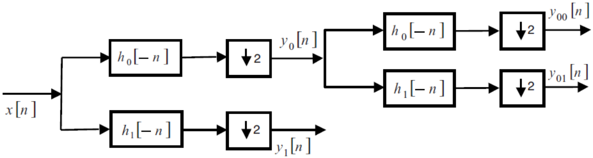
$$x[n] = \frac{1}{\sqrt{N}} \sum_k W_\phi(j_0, k)\phi_{j_0,k}[n] + \frac{1}{\sqrt{N}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j, k)\psi_{j,k}[n]$$

Egymásba ágyazott Haar-wavelet és skálafüggvények

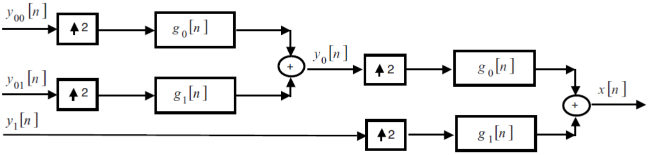


$$\mathcal{V}_3 = \mathcal{V}_0 \oplus \mathcal{W}_0 \oplus \mathcal{W}_1 \oplus \mathcal{W}_2$$

Oktávsávos részsávos DWT

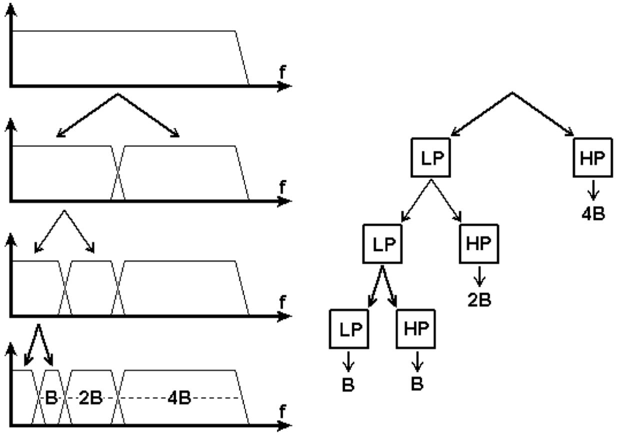


(a)

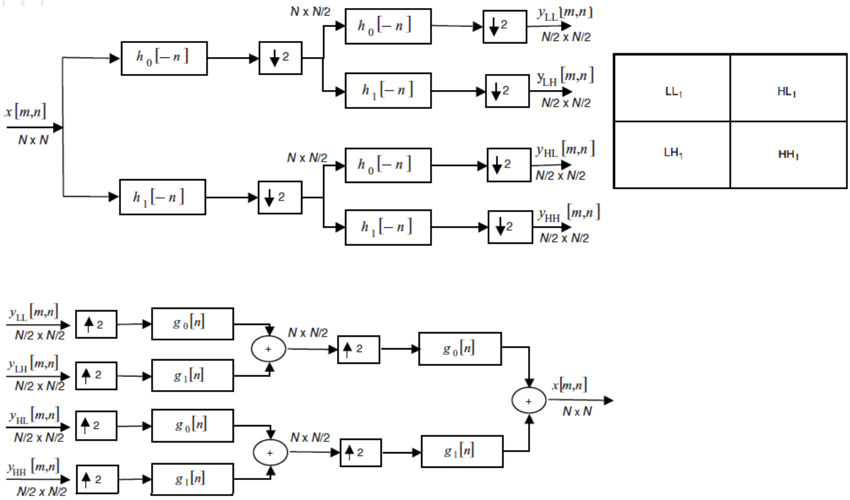


(b)

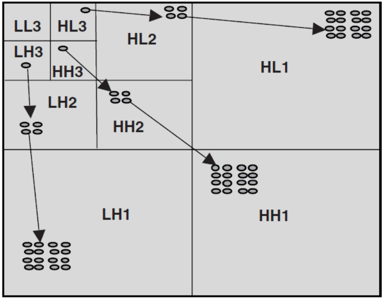
Frekvenciatartománybeli értelmezés



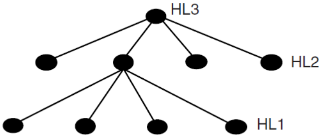
2D DWT


x[m,n]
N x N
h0[−n]
↓2
N x N/2
h0[−n]
↓2
yLL[m,n]
N/2 x N/2
h1[−n]
↓2
yLH[m,n]
N/2 x N/2
h1[−n]
↓2
N x N/2
h0[−n]
↓2
yHL[m,n]
N/2 x N/2
h1[−n]
↓2
yHH[m,n]
N/2 x N/2
LL1
HL1
LH1
HH1
yLL[m,n]
N/2 x N/2
↑2
g0[n]
yLH[m,n]
N/2 x N/2
↑2
g1[n]
+
N x N/2
↑2
g0[n]
yHL[m,n]
N/2 x N/2
↑2
g0[n]
yHH[m,n]
N/2 x N/2
↑2
g1[n]
+
N x N/2
↑2
g1[n]
+
x[m,n]
N x N

Zero-tree feltételezés



(a)



(b)

EZW SAQ (Succesive Approximation Quantization)

63	-34	49	10	7	13	-12	7
-31	23	14	-13	3	4	6	-1
15	14	3	-12	5	-7	3	9
-9	-7	14	8	4	-2	3	2
-5	9	-1	47	4	6	-2	2
3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	11	5	6	0	3	-4	4

EZW SAQ (Succesive Approximation Quantization)