

BASIC NOTIONS OF ACOUSTICS

*Study aid for learning of Communications Acoustics
VIHIM 000*

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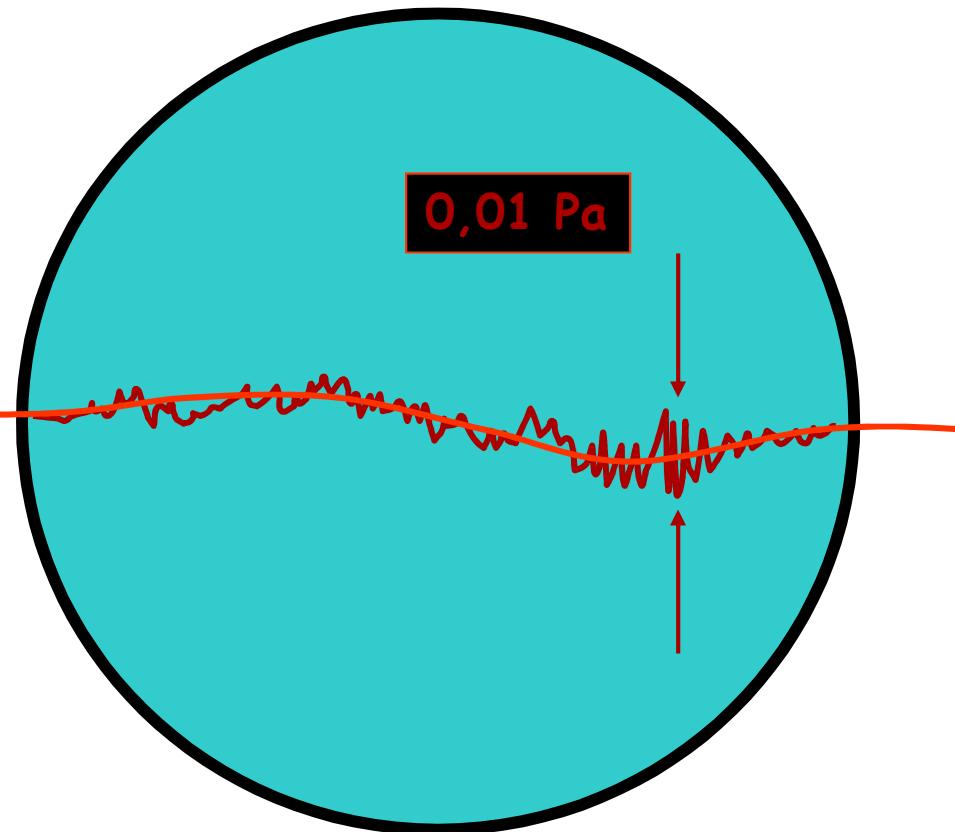
What is sound?

- Static (\approx constant) pressure of air



The sound is...

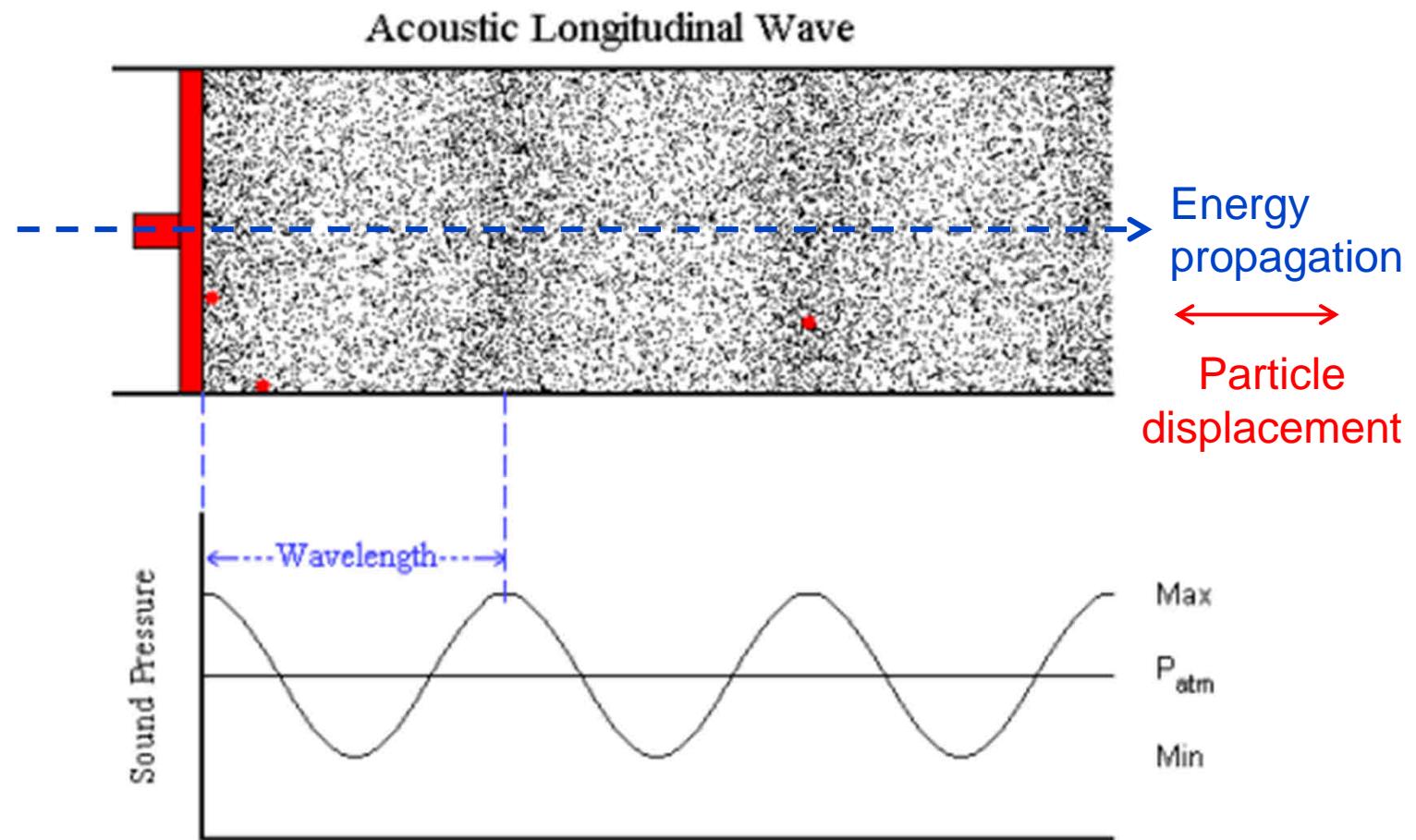
- The dynamic pressure fluctuation of air (water, etc.)



100000 Pa

How are sounds generated??

- Pressure irregularities of fast flowing air stream
- Density fluctuation of air, caused by vibrating surfaces





Sound pressure

The notion of sound pressure:

dynamic force acting on a unity area

unit: Pascal = N/m²

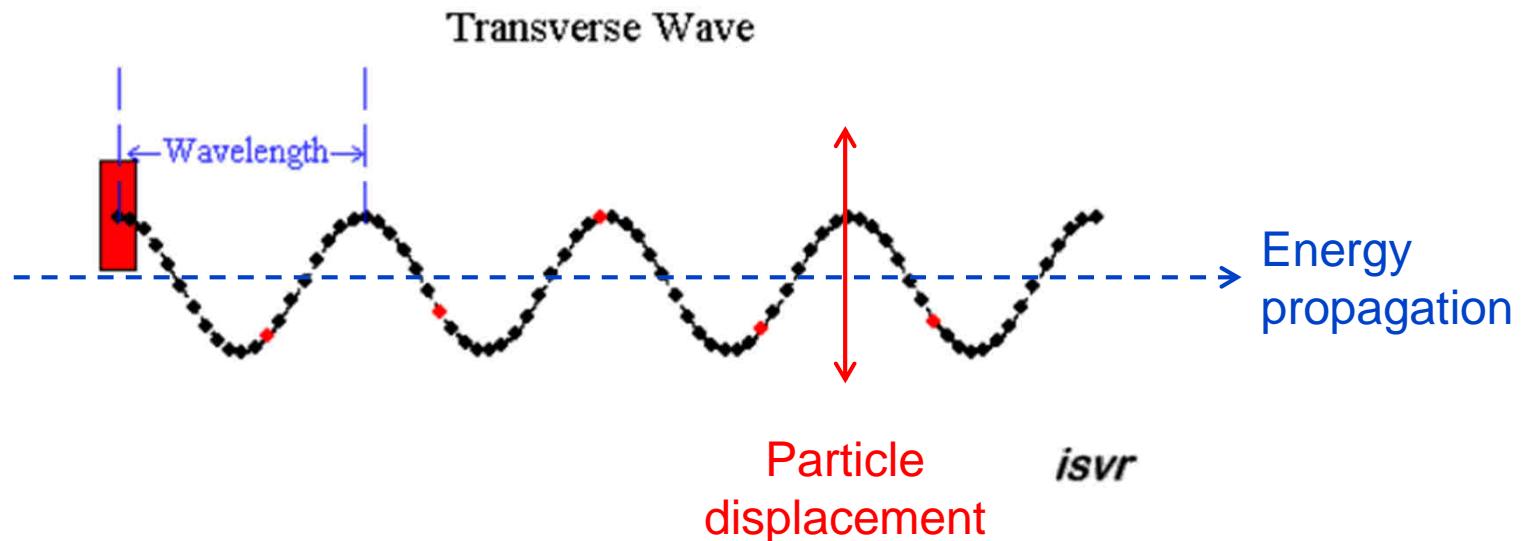
notation: *p*



Blaise Pascal, 1623 - 1662

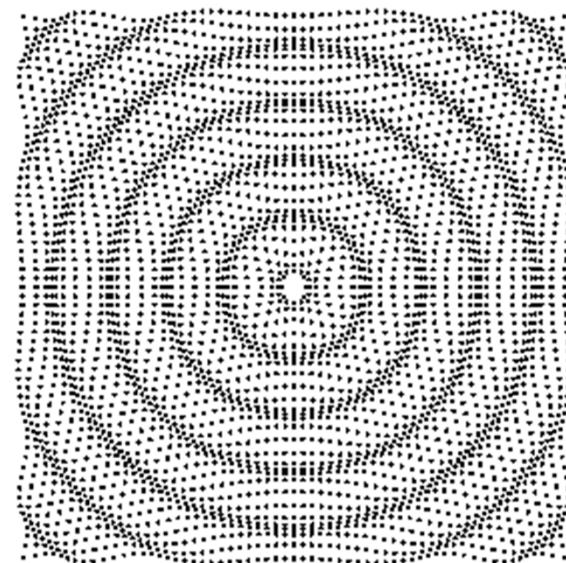
Vibrations of string

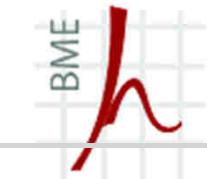
- Transversal waves



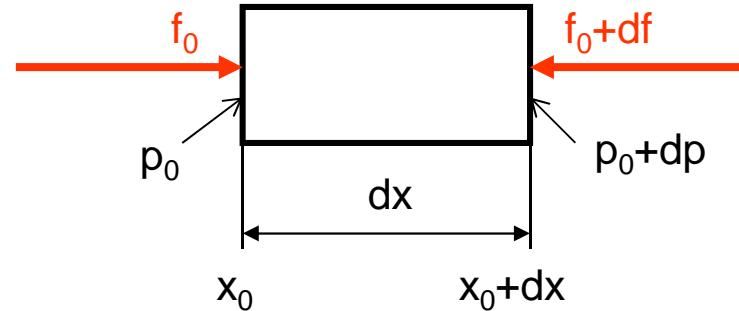


Spherical waves, generated by a point source





The 1st basic equation of sound field



$$F = ma$$

Newton's 2nd law

$$\sum F = \frac{\partial}{\partial t} (mv)$$

In a more general form: the impulse theorem

$$f|_{x_0} - f|_{x_0+dx} = A \left(p|_{x_0} - p|_{x_0+dx} \right) = A \left(p|_{x_0} - \left[p|_{x_0} + \frac{\partial p}{\partial x} dx \right] \right) = -A \frac{\partial p}{\partial x} dx$$

$$\frac{\partial}{\partial t} (mv) = \frac{\partial}{\partial t} (V \rho v) = \frac{\partial}{\partial t} (A dx \rho v) = A dx \frac{\partial}{\partial t} (\rho v)$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial p}{\partial x} = 0$$

Momentum,
or equilibrium,
or Euler equation

The notion of flow rate



$$\varphi = \frac{1}{A} \frac{dm}{dt} = \frac{1}{A} \frac{(Adx)\rho}{dt} = \rho v$$

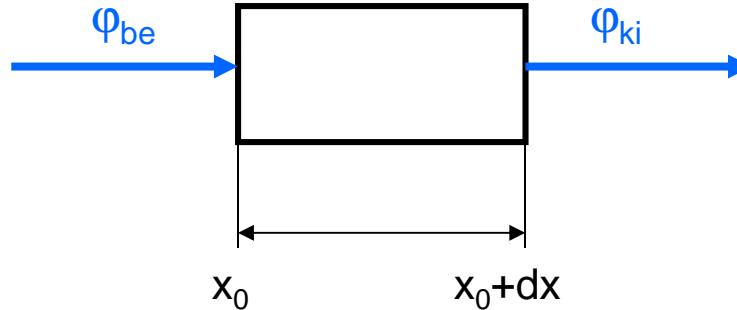
Leonhard Euler, 1707 - 1783

The amount of material flowing through unity surface per unit time: **flow rate**

The amount of material flowing through the surface A : flow rate \times surface

$$\frac{dm}{dt} = A(\rho v)$$

The 2nd basic equation of sound field



The net amount of material, i.e. the difference of material flowing in and out per unit time:

$$\Delta A(\rho v) = A(\rho v)|_{x_0} - A(\rho v)|_{x_0+dx} = A(\rho v)|_{x_0} - \left(A(\rho v)|_{x_0} + A \frac{\partial(\rho v)}{\partial x} dx \right) = -A \frac{\partial(\rho v)}{\partial x} dx$$

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} (A dx \rho) = A dx \frac{\partial \rho}{\partial t} = -A \frac{\partial(\rho v)}{\partial x} dx$$

$$\frac{\partial(\rho v)}{\partial x} + \frac{\partial \rho}{\partial t} = 0$$

Continuity equation or
equation of mass
conservation



On the way to the wave equation

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial(\rho v)}{\partial x} + \frac{\partial \rho}{\partial t} = 0$$

$$p = p_{st} + p'$$

$$\rho = \rho_{st} + \rho'$$

$$v = v_0 + v'$$

$$\rho_{st} \frac{\partial v'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \quad \frac{\partial}{\partial x}$$

$$\rho_{st} \frac{\partial v'}{\partial x} + \frac{\partial \rho'}{\partial t} = 0$$

Relationship between density and pressure: $\rho' = p' \frac{\rho_{st}}{\kappa p_{st}}$

$$\begin{aligned} p' &\triangleq p \\ v' &\triangleq v \end{aligned}$$

$$\rho_{st} \frac{\partial v'}{\partial x} + \frac{\partial}{\partial t} \left(p' \frac{\rho_{st}}{\kappa p_{st}} \right) = 0 \quad \frac{\partial}{\partial t}$$

$$\rho_{st} \frac{\partial^2 v}{\partial t \partial x} + \frac{\partial^2 p}{\partial x^2} = 0$$

$$\rho_{st} \frac{\partial^2 v}{\partial x \partial t} + \frac{\rho_{st}}{\kappa p_{st}} \frac{\partial^2 p}{\partial t^2} = 0$$



Terminal point of the derivation:

$$\rho_{st} \frac{\partial^2 v}{\partial t \partial x} + \frac{\partial^2 p}{\partial x^2} = 0$$

$$-\rho_{st} \frac{\partial^2 v}{\partial x \partial t} + \frac{\rho_{st}}{\kappa p_{st}} \frac{\partial^2 p}{\partial t^2} = 0$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{\sqrt{\frac{\kappa p_{st}}{\rho_{st}}}} \frac{\partial^2 p}{\partial t^2} = 0$$

$$c = \sqrt{\frac{\kappa p_{st}}{\rho_{st}}}$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

One-dimensional
wave equation
(d'Alambert, 1747)



Solution of the wave equation

Any function, in which the independent variables x (space) and t (time) appear in the following combination:

$$p(x, t) = f\left(t - \frac{x}{c}\right)$$





And what if we are in 3D space?

$$\nabla^2 p(x, y, z, t) - \frac{1}{c^2} \frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0$$

The simplest solution is obtained in spherical coordinate system, where there is just one spatial variable r , which is the distance from the origin :

$$\frac{\partial^2 (rp)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 (rp)}{\partial t^2} = 0$$

Solution:

$$p(r, t) = \frac{const}{r} f\left(t - \frac{r}{c}\right)$$

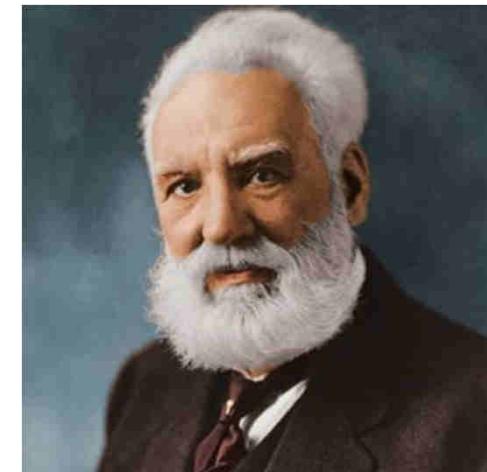
The notion of sound pressure level (decibel)

Notion of sound pressure **level**:

$$L_p = 10 * \log_{10} \left(\frac{p^2}{p_0^2} \right) = 20 * \log_{10} \left(\frac{p}{p_0} \right) \text{ [dB]}$$

Remarks:

- The SPL is a relative (referenced) quantity + a logarithmic transformation
- the SPL is essentially a power-type quantity
- p_0 corresponds to the minimum audible sound



Alexander Graham Bell, 1847 - 1922

The notion of frequency

- Periodic effects: regular repetition of a process or phenomenon
- Measure is the time of repetition, T
- Frequency: number of periodic phenomena per unit time, f
- Time and frequency are inverse quantities: the longer the time of period, the lower the frequency
- Unit of frequency: Hertz (Hz), 1/s

$$f = \frac{1}{T}$$

- Relevant frequencies in acoustics:
20 Hz – 20 kHz
- Radio waves: kHz → MHz → GHz



Heinrich Rudolph Hertz, 1857-1894

The notion of wavelength

$$\lambda = \frac{c}{f} = \frac{340}{f}$$

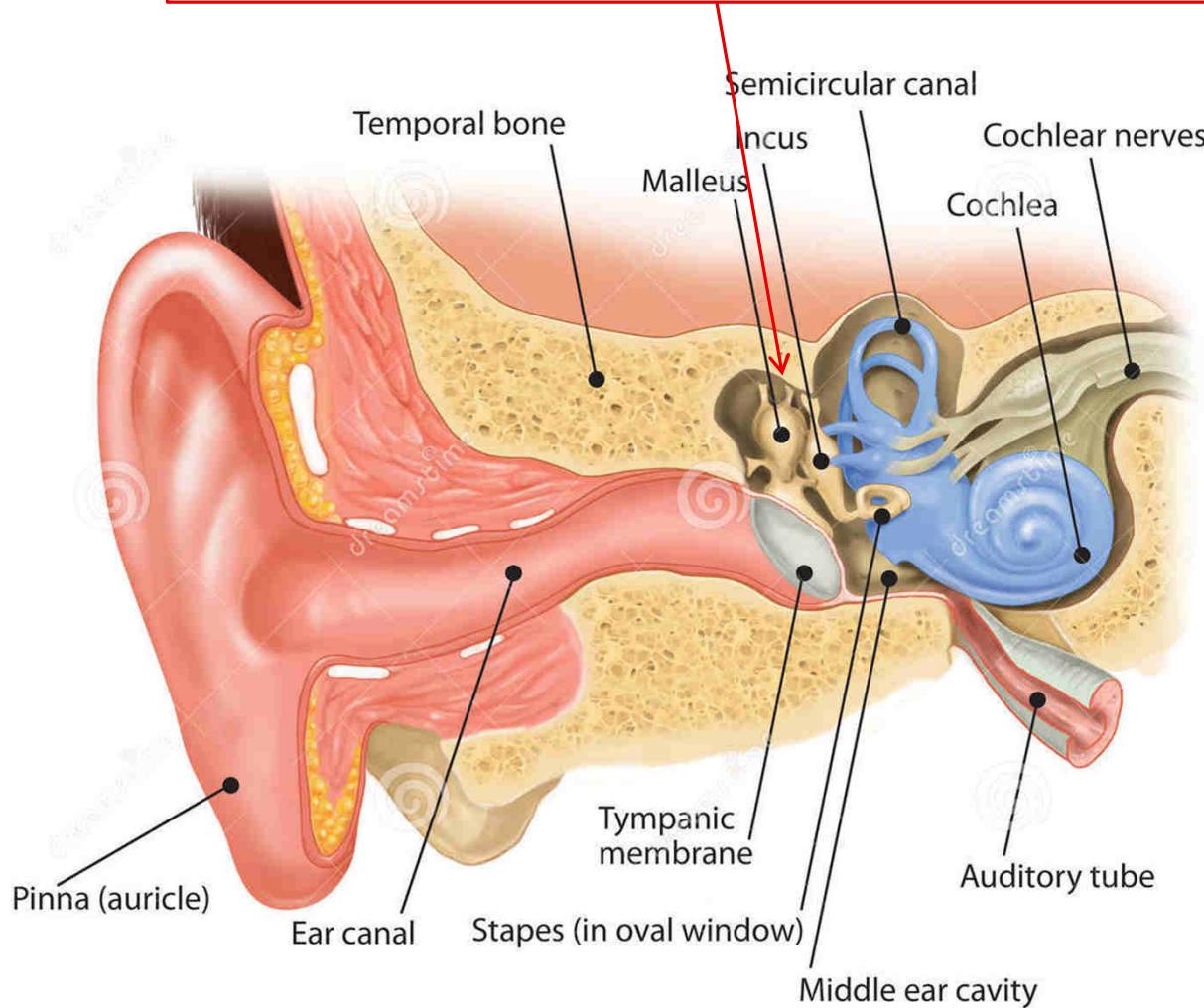
- Reciprocal quantity of the frequency:
the higher the frequency, the lower the wavelength
- 20 Hz to 20 kHz corresponds to 17 m to 17 mm
- Wave phenomena take place if $\lambda \approx d$
 - always and everywhere in our daily life!
- $C_{\text{electromagnetic}} \approx 10^6 \times C_{\text{sound}}$

$$f_{\text{microwave}} \approx 10^6 \times f_{\text{sound}}$$

wavelengths are commensurable

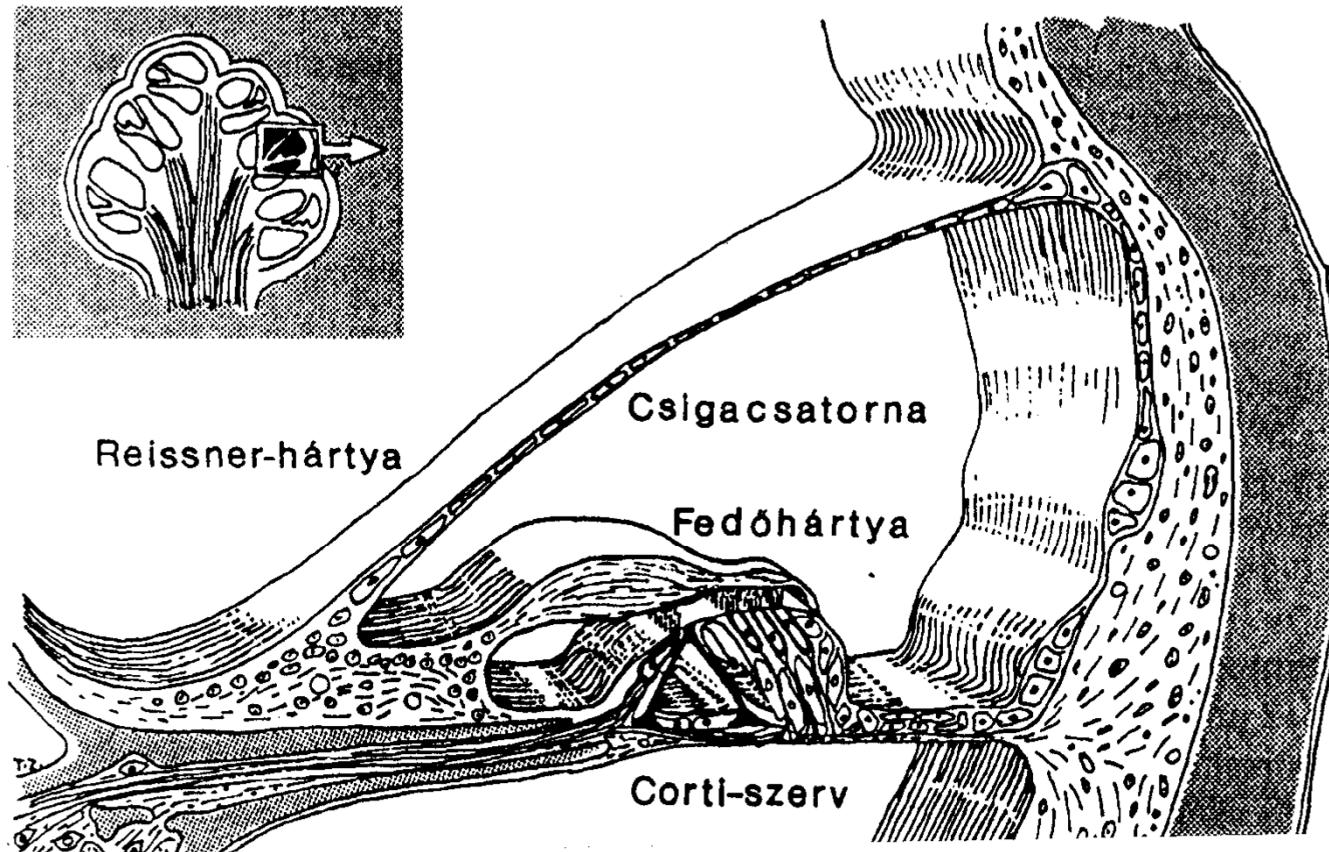
How do we hear? The human ear

malleus, incus, stapes = hammer, anvil and stirrup

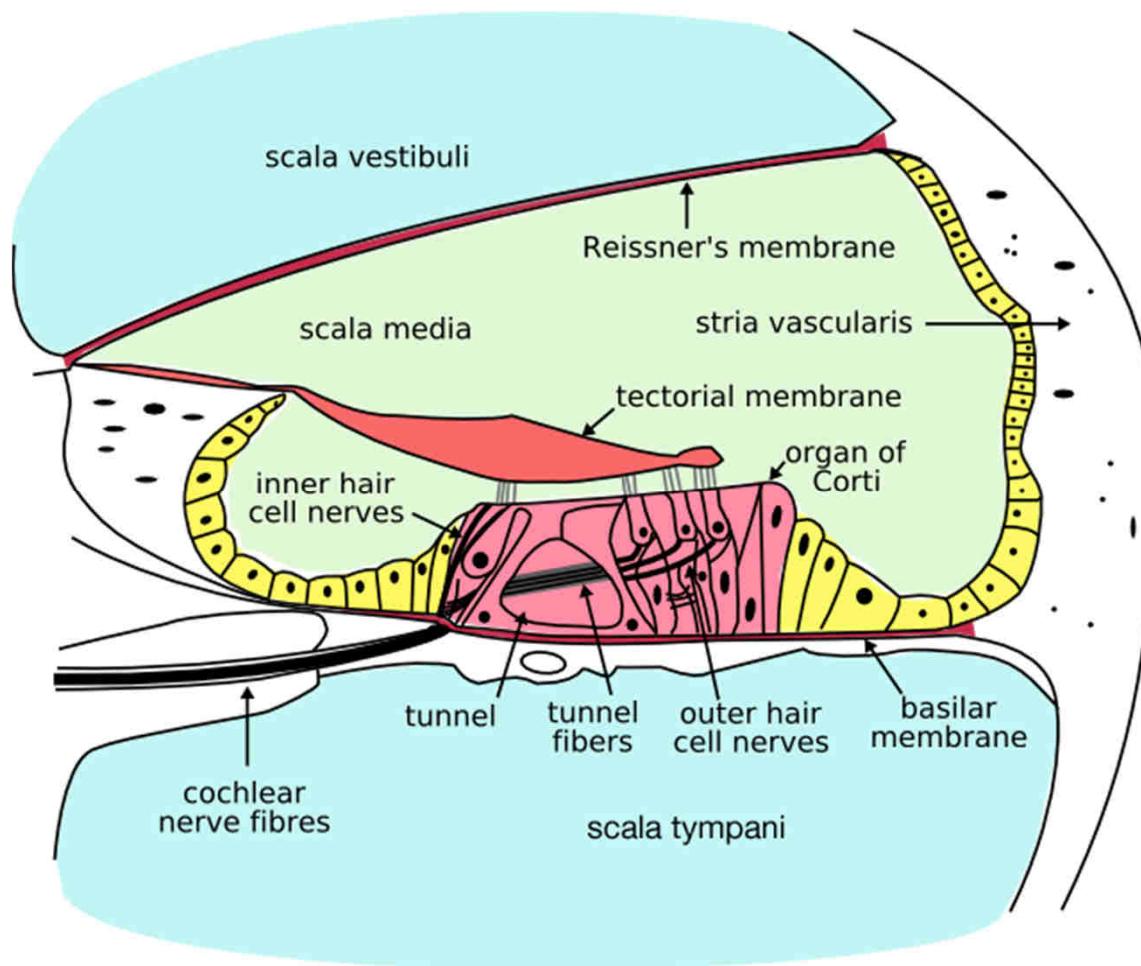


The Corti-organ

A csiga egyik menetének metszete. Közel anatómiai hűségű ábra. A csarnoki és a dobúri csatorna csak részben látszik, a csatornarendszer természetesen körben zárt. Az idegek bal felé távoznak a csigából, és ott találhatók az idegvezetékek első sejtmagjai.



Demonstration of operation of the human ear



- <https://www.youtube.com/watch?v=46aNGGNPm7s>

Inner ear infections

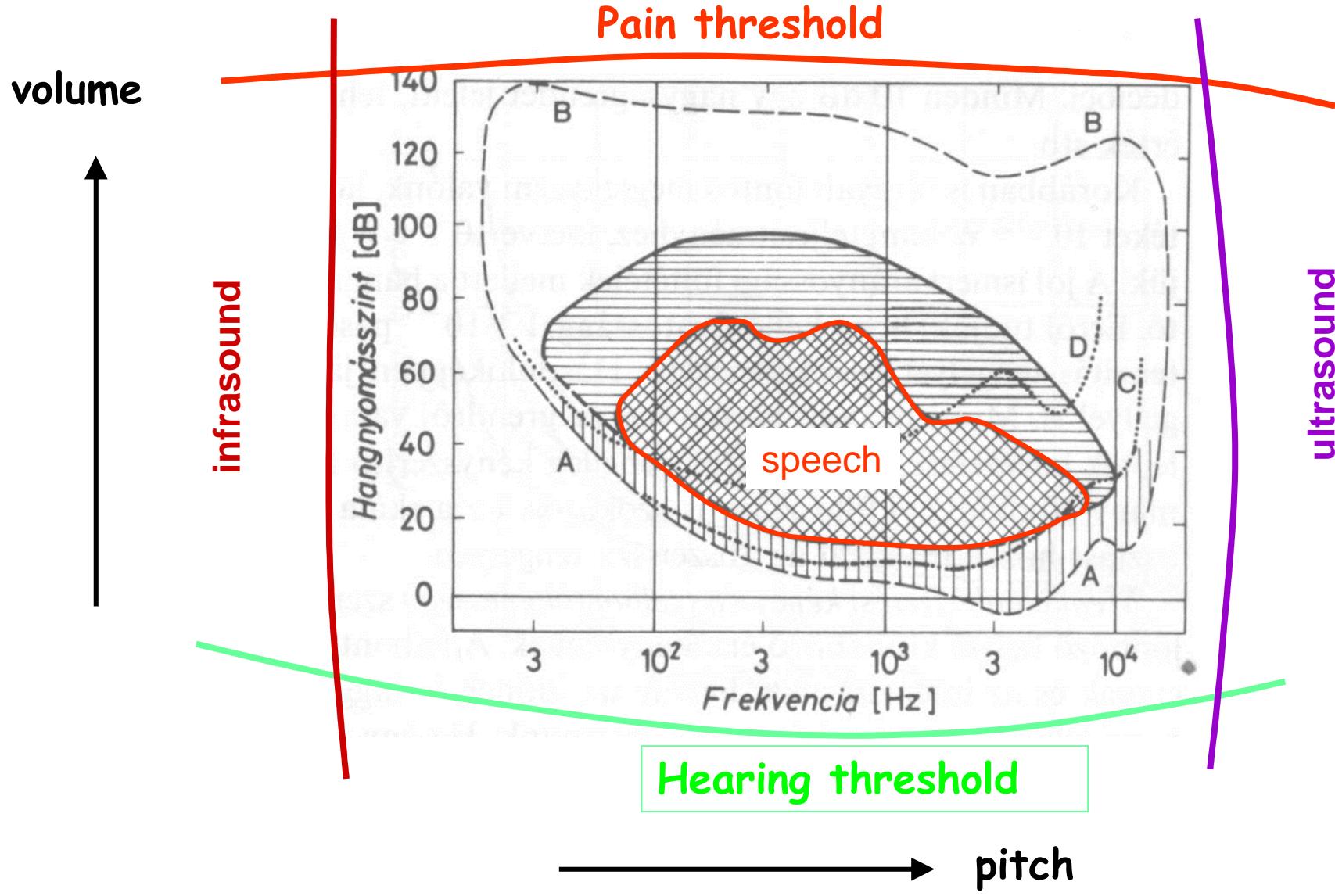


A visit to the doctor's may be a good idea anyway,

https://www.youtube.com/watch?annotation_id=annotation_671624879&feature=iv&src_vid=WOgl5IAR_bQ&v=ZJFc9gdB75w

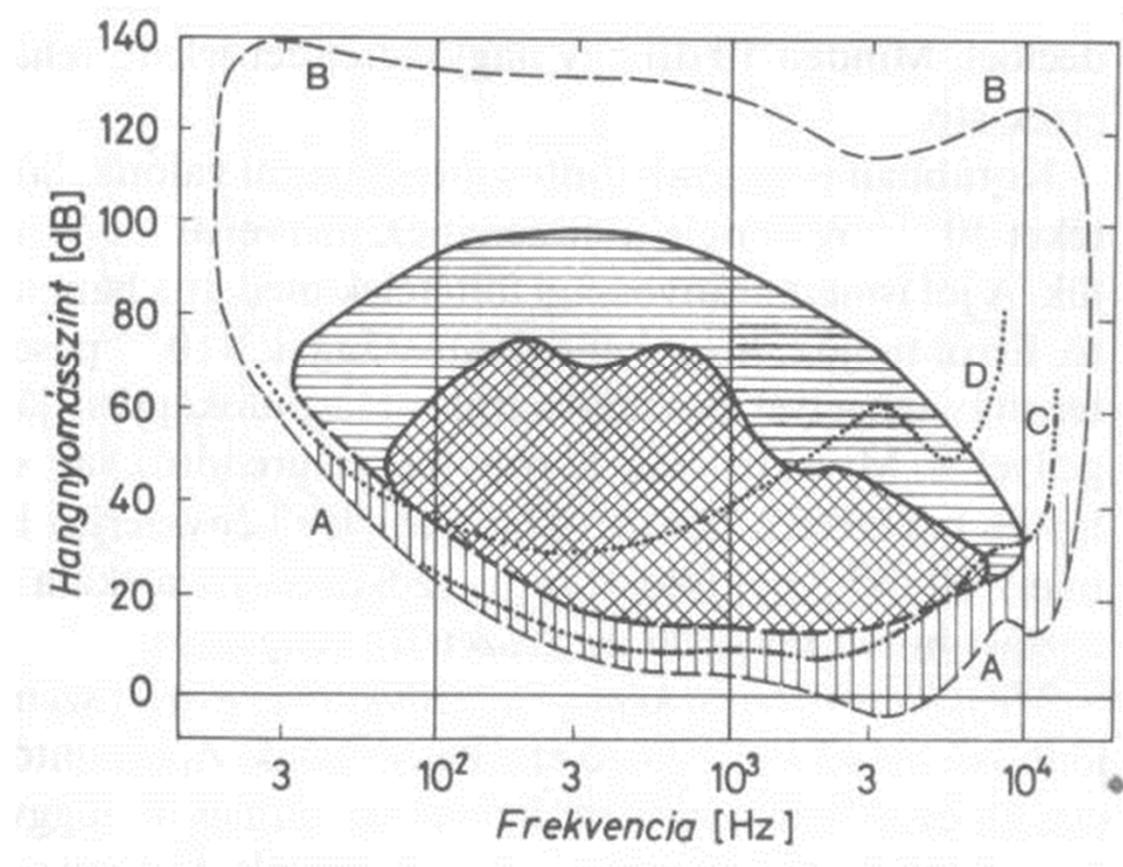


What do we hear? The limits of human hearing

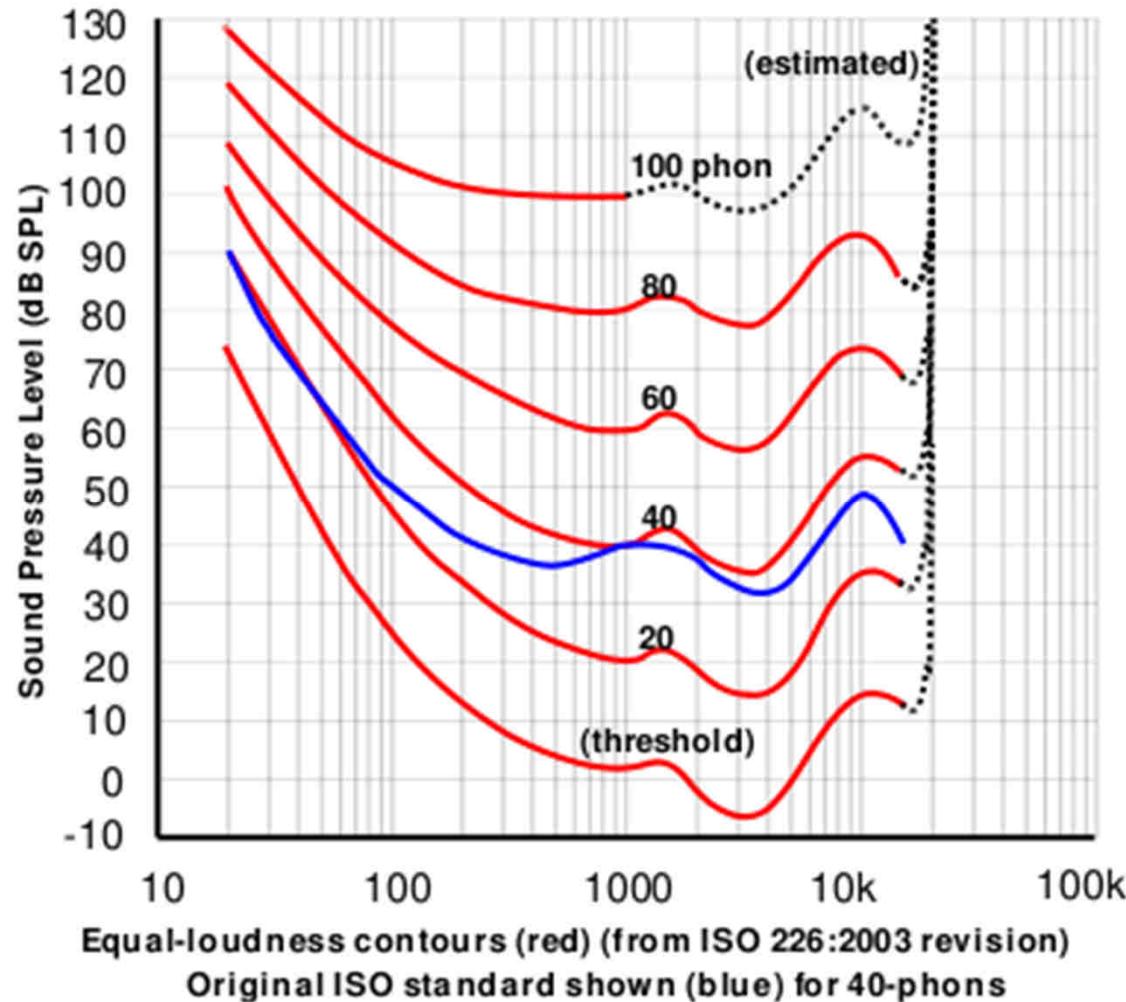


Hearing area, more precisely

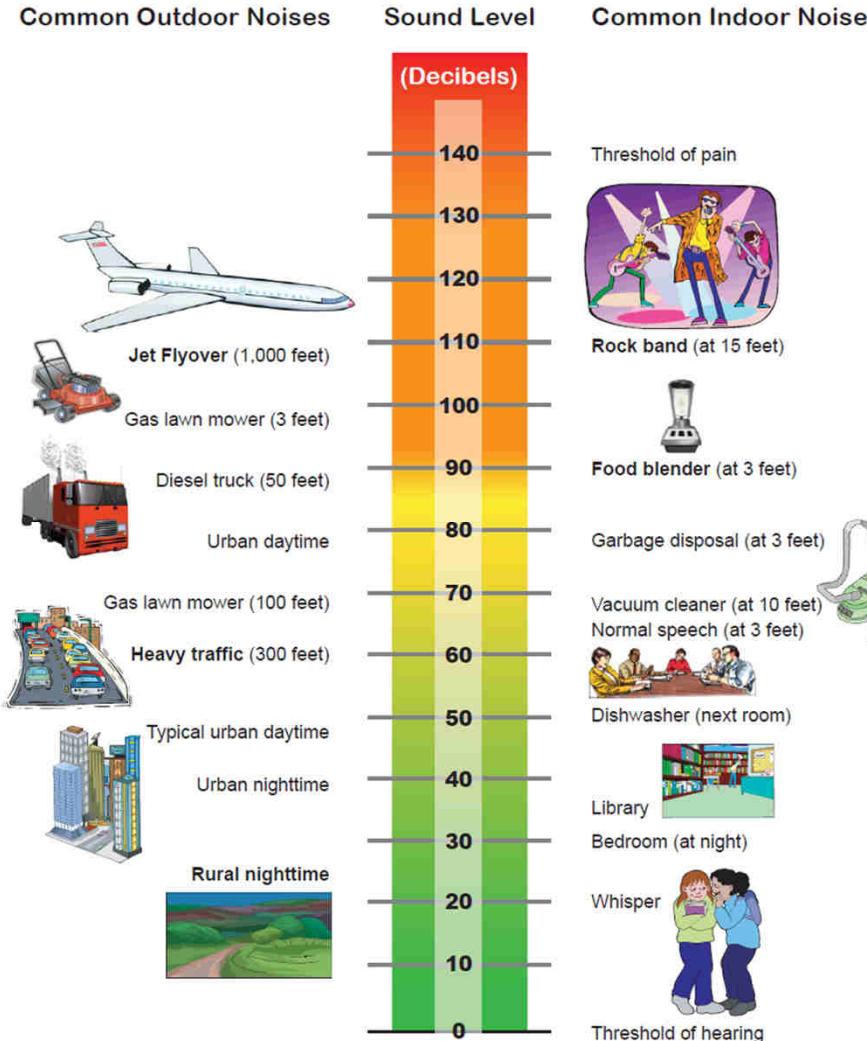
- AA: hearing threshold
- BB: pain threshold
- C: hearing of a 55-y old healthy man
- D: hearing of an industrial worker with impaired hearing
- Dashed line: background noise of concert hall



The Fletcher-Munson curves of equal loudness



Typical sound pressure levels



Relationships of impact and sensation

Psychic response

$$\Psi = f(\Phi)$$

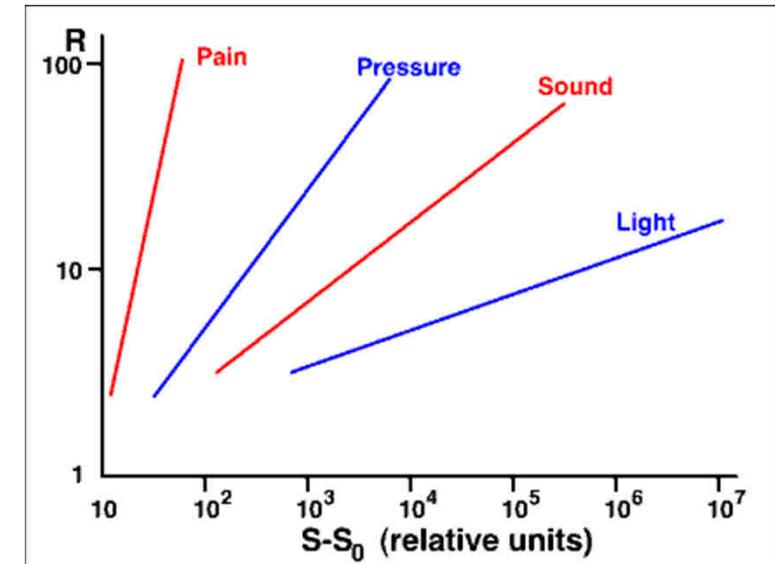
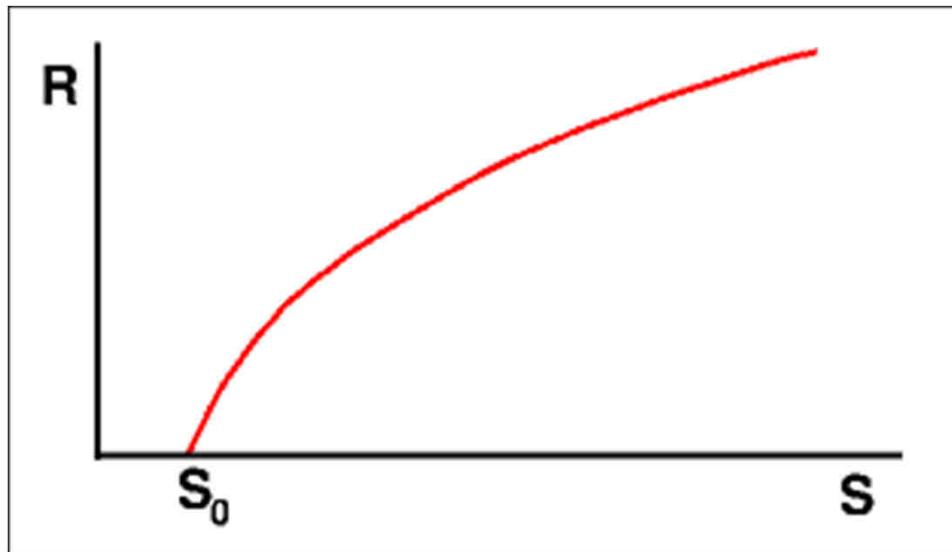
Physical impact

- Weber-Fechner law:

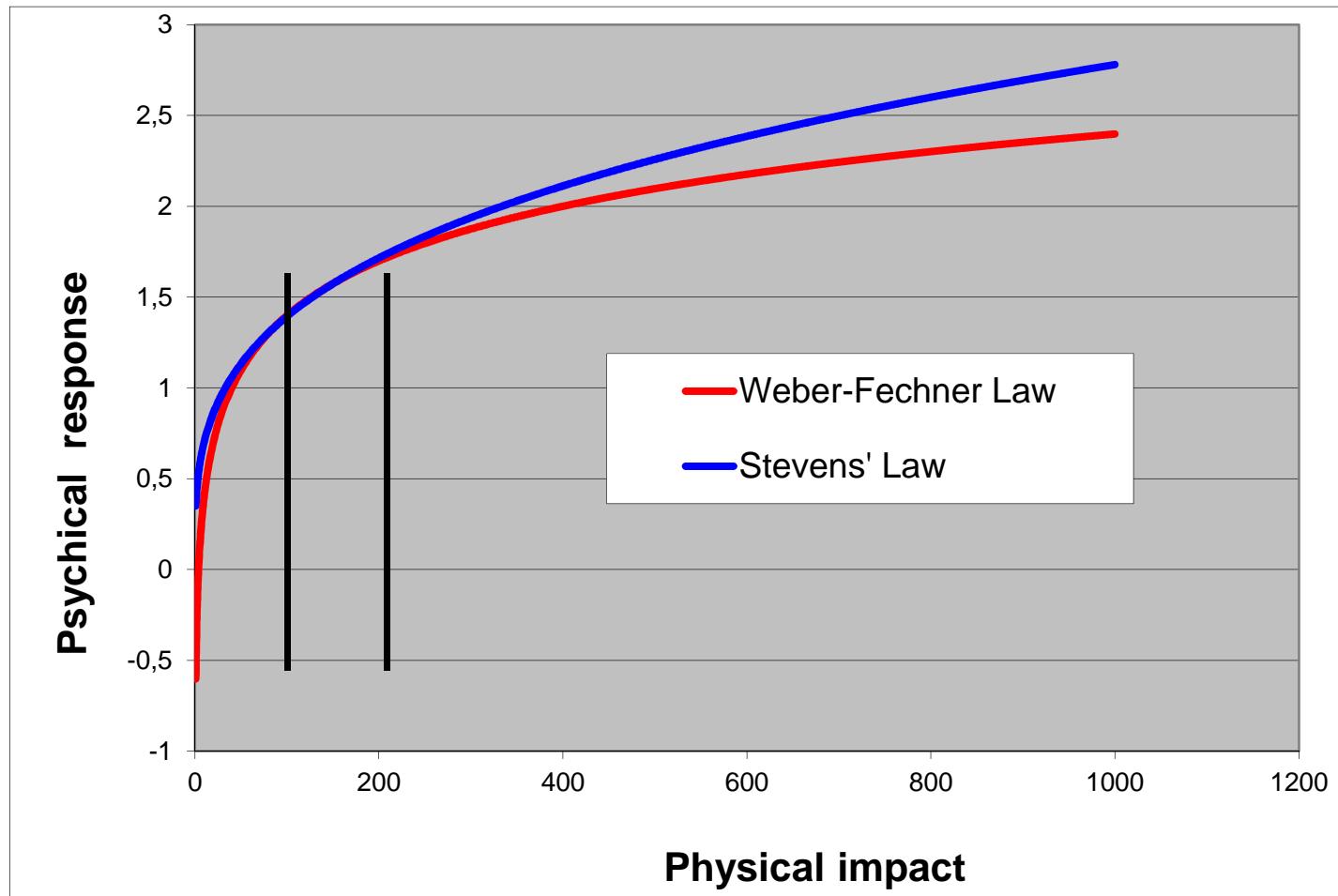
$$\Psi \approx \log(\Phi)$$

- Stevens law:

$$\Psi \approx \Phi^n$$



Comparison of laws of sensational response





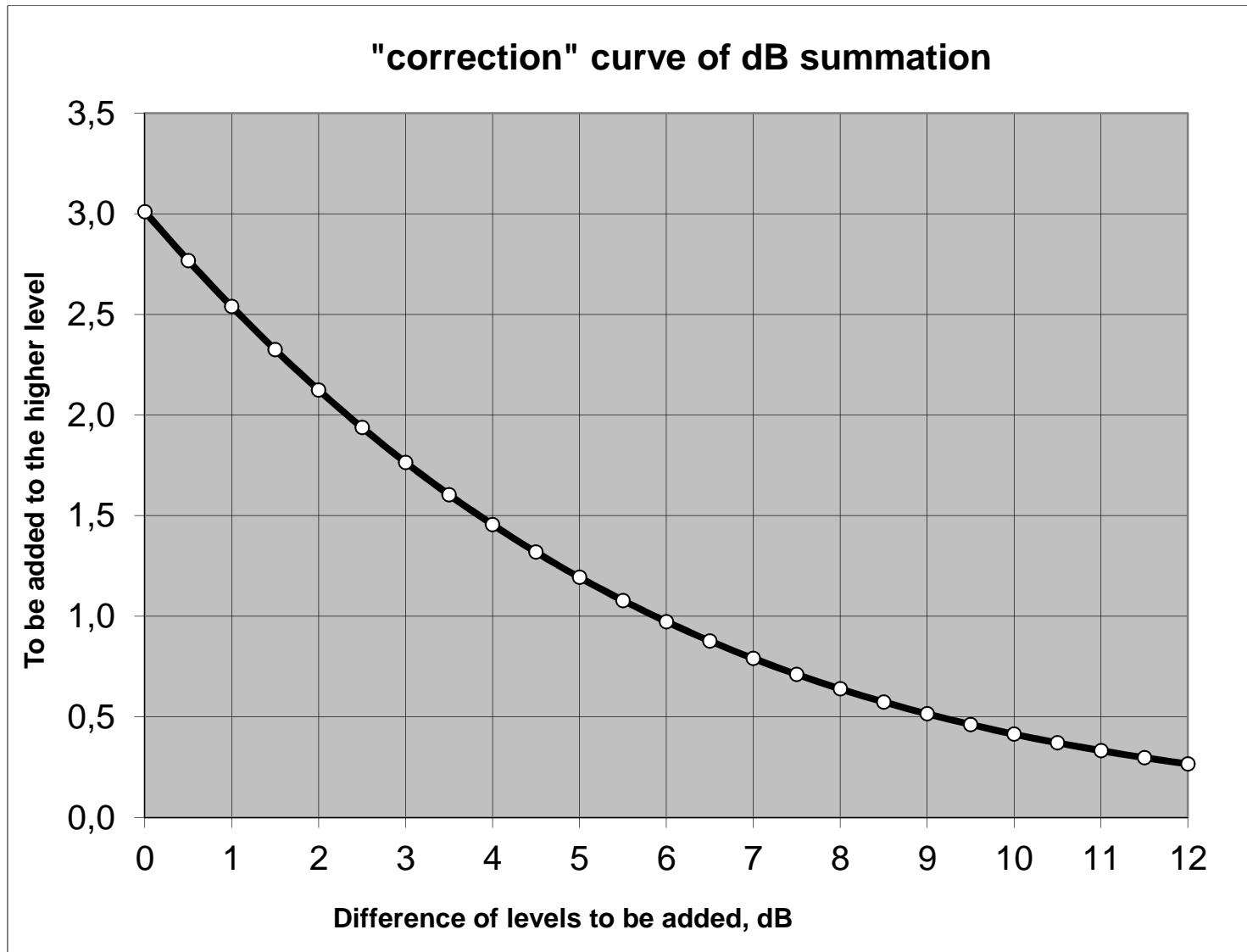
Operations with decibel

- What is possible to sum up directly?
 - **decibels?:? NO!!!**
 - only **power quantities**, i.e. the square of sound pressures!

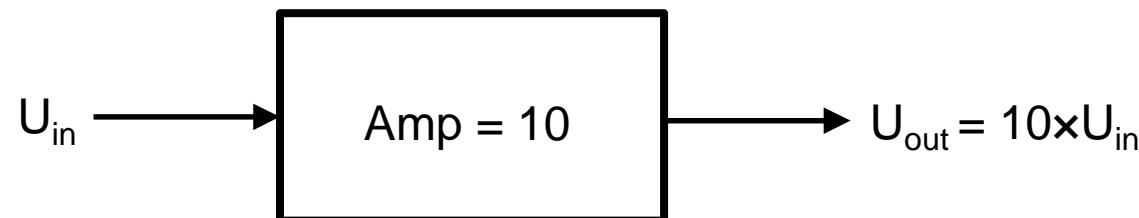
- Ways of dB calculation:
 - Step by step, by means of a calculator
 - graphically
 - by means of tables
 - by computer



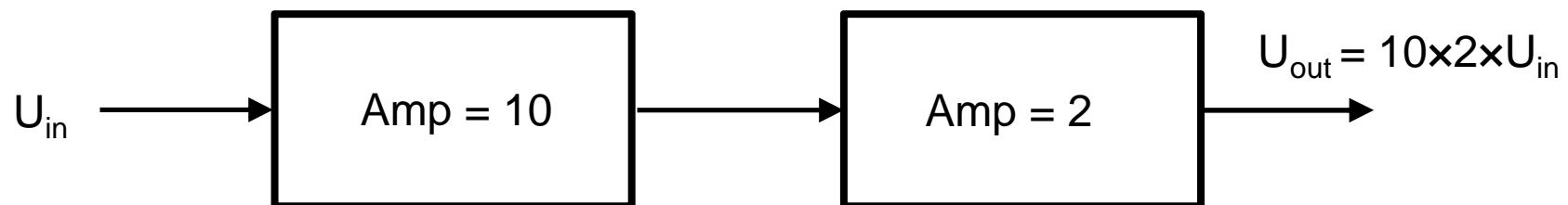
Summation of SPLs



Other uses of dB: amplification and reduction



$$A = 20 \times \log_{10} \frac{U_{out}}{U_{in}} = 20 \times \log_{10} 10 = 20 \text{ dB}$$



$$A = 20 \times \log_{10} \frac{U_{out}}{U_{in}} = 20 \times \log_{10} (10 \times 2) = 20 + 6 = 26 \text{ dB}$$