The application of the Wave Based Technique using a boundary error indicator controlled adaptive strategy

Tamás Mócsai, mocsai@hit.bme.hu

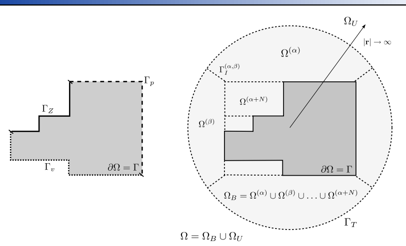
BME Department of Telecommunications

Laboratory of Acoustics and Studio Technologies

Overview

In the recent years the Wave Based Technique (WBT) has become a prospective and efficient alternative of the already well established element based acoustic simulation methods (FEM, BEM). The WBT is based on an indirect Trefftz approach, as it uses exact solutions of the governing differential equation, requiring a less fine element discretization. The resulted smaller numerical models exhibit an enhanced computational efficiency, thus the frequency range of the calculation can be extended towards the mid-frequencies. The aim of the current research is the development of a WBT calculation tool, which requires less user experience and control to set up the numerical model, by means of a novel boundary-error-indicator controlled adaptive strategy, which provides accuracy control and a-posteriori error monitoring. The practical application of the strategy is presented on an industrial-like simplified model of a car combustion engine.

Problem definition

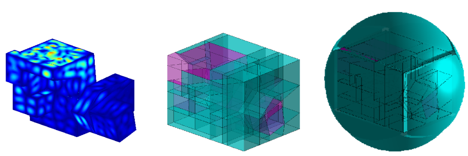


The unbounded Helmholtz problem

$$\nabla^2 p(\mathbf{r}) + k^2 p(\mathbf{r}) = 0$$

- Pressure (Dirichlet) boundary condition $\bar{p}(\mathbf{r})$ at Γ_p: $p(\mathbf{r}) = \bar{p}(\mathbf{r})\ \mathbf{r} \in \Gamma_p$
- Normal velocity (Neumann) boundary condition $\bar{v}_n(\mathbf{r})$ at Γ_v: $v_n(\mathbf{r}) = \bar{v}_n(\mathbf{r})\ \mathbf{r} \in \Gamma_v$
- Normal impedance (Robin) boundary condition $\bar{Z}_n(\mathbf{r})$ at Γ_Z: $Z_n(\mathbf{r}) = \bar{Z}_n(\mathbf{r})\ \mathbf{r} \in \Gamma_Z$
- The Sommerfeld-radiation condition: $\lim_{|\mathbf{r}|\to\infty}\left[|\mathbf{r}|\left(\frac{\partial p(\mathbf{r})}{\partial|\mathbf{r}|} + jkp(\mathbf{r})\right)\right] = 0$

Model description



The simplified engine geometry ($0.746\ m \times 0.561\ m \times 0.585\ m$) is filled up with 33 wave domains up to the spherical truncation boundary Γ_T. The engine boundary has been modeled as an aluminium FEM plate mesh (d=$2mm$), excited by two anti-phase point forces on the top plate. The FE velocity distribution (2000 Hz depicted) has been applied as normal velocity boundary condition on the radiating acoustic boundary, which represents a one-way structural-fluid acoustic coupling (the back-coupling from the fluid onto the structure is omitted).

The Wave Based Technique

In order to tackle problems in unbounded domains, an artificial truncation boundary Γ_T is introduced. The problem domain is thus divided into a bounded region Ω_B and an unbounded region Ω_U. The steady-state dynamic acoustic pressure field $p^{(\alpha)}(\mathbf{r})$ is approximated using the following pressure expansion within a subdomain $\Omega^{(\alpha)}$:

$$p^{(\alpha)}(\mathbf{r}) \approx \hat{p}^{(\alpha)}(\mathbf{r}) = \sum p_w^{(\alpha)} \phi_w^{(\alpha)}(\mathbf{r}) = \mathbf{\Phi}^{(\alpha)}(\mathbf{r})\mathbf{p_w}^{(\alpha)}$$

For the pressure expansion in the bounded part Ω_B the following T-complete set is used:

$$\phi_w^{(\alpha)}(\mathbf{r}_{(x,y,z)}) = \begin{cases} \phi_{w_r}^{(\alpha)}(x,y,z) = \cos(k_{x_{wr}}^{(\alpha)} x)\cos(k_{y_{wr}}^{(\alpha)} y)\exp^{-jk_{z_{wr}}^{(\alpha)} z} \\ \phi_{w_s}^{(\alpha)}(x,y,z) = \cos(k_{x_{ws}}^{(\alpha)} x)\exp^{-jk_{y_{ws}}^{(\alpha)} y}\cos(k_{z_{ws}}^{(\alpha)} z) \\ \phi_{w_t}^{(\alpha)}(x,y,z) = \exp^{-jk_{x_{wt}}^{(\alpha)} x}\cos(k_{y_{wt}}^{(\alpha)} y)\cos(k_{z_{wt}}^{(\alpha)} z) \end{cases}$$

A valid basis set, inherently satisfying the Sommerfeld-condition for the expansion in Ω_U is given in spherical coordinates:

$$p^{(\Omega_U)}(r,\phi,\theta) = \sum_{l=0}^{n_{RF}} \sum_{m=-l}^{l} p_{lm} h_l(kr) Y_{lm}(\phi,\theta)$$

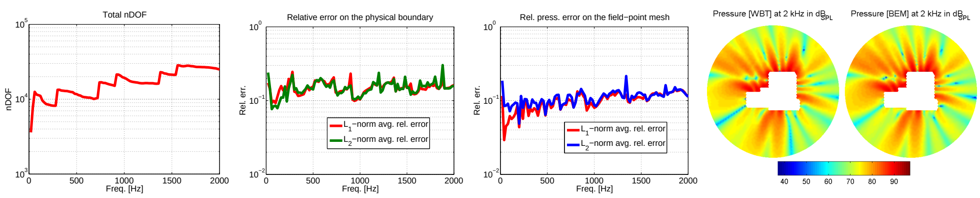
which are the Spherical harmonic functions, defined as combinations of the Legendre-polynomials and Hankel-functions of the second kind.

Boundary-error-indicator based adaptive control

An advantage of the WBT over FEM/BEM is that the domain subdivision (h-refinement) is kept fixed over the frequency range, and the model refinement process only involves the enlargement of the wave function set (p-refinement). Utilizing this advantage, low-complexity, but highly efficient dynamic refinement strategies can be adopted. In the indirect Trefftz formulation, the solution vector $\mathbf{p_w}^{(\alpha)}$ is determined so that the residuals of the boundary conditions (physical and artifical continuity conditions between the subdomains) are minimized simultaneously for each domain $\Omega^{(\alpha)}$. In the WBT the Galerkin approach is applied: for each subdomain $\Omega^{(\alpha)}$, the error functions are orthogonalized with respect to the weighting function $\widetilde{p}^{(\alpha)}$ or its derivative. As the Helmholtz-problem along with the specified boundary conditions is a well-posed problem, the continuous dependence of the solution on the boundary data is ensured. The Trefftz approach ensures that the differential equation is exactly satisfied, while the solution is enforced to satisfy the boundary conditions in a weighted residual integral sense. By monitoring the a-posteriori boundary residual error, the size of the wave function set can be controlled to bound the approximation error. The proposed error indicator evaluates the approximation error on the physical boundary and the continuity error between the subdomains, requiring the evaluation of boundary integrals only. The approximation error on the physical boundary parts Γ_p, Γ_v and Γ_Z expresses the deviation of the WBT solution from the prescribed boundary conditions, for the pressure velocity and normal impedance respectively. To serve as an error indicator the L_2 norm value of these boundary errors is taken, expressed in a relative sense with respect to average L_2 norm values of the corresponding physical quantites over the corresponding surface parts. Hence, the indicator values are independent on the magnitude of the the associated quantities (ie. the input power of a radiating model) and the interpretation of the indicator values follows a general engineering approach. The error indicators for the continuity errors between the subdomains express the difference beetween the pressure and velocity distribution in relative sense on the common coupling interface of the subdomains. Both the error indicators for the physical boundary conditions, and for the continuity conditions are defined locally and globally. Local indicators indicate subdomain-wise boundary errors, while global indicators indicate the boundary errors in an average L_2-sense for the whole numerical model . The local and global error indicators then allow the implementation of an adaptive calculation scheme to keep the boundary error bounded by prescribed threshold levels, which is realised by an adaptive wave function refinement-recalculation scheme.

Numerical results

For the assesment of the approach, the boundary error has been evaluated as a total relative velocity error on the engine boundary. The field pressure error compared to a reference Direct BEM solution ($h = 8mm$, $n_{DOF} = 24242$) has also been evaluated. L_1- and L_2-norm results obtained by the adaptive strategy with a prescribed error tolerance of 0.2:



The presented flexible, robust adaptive calculation approach was able to keep the boundary errors of a WBT model bounded by the predefined threshold. The avarege pressure amplitude prediction error on the depicted contour field was ca. 2 dB.