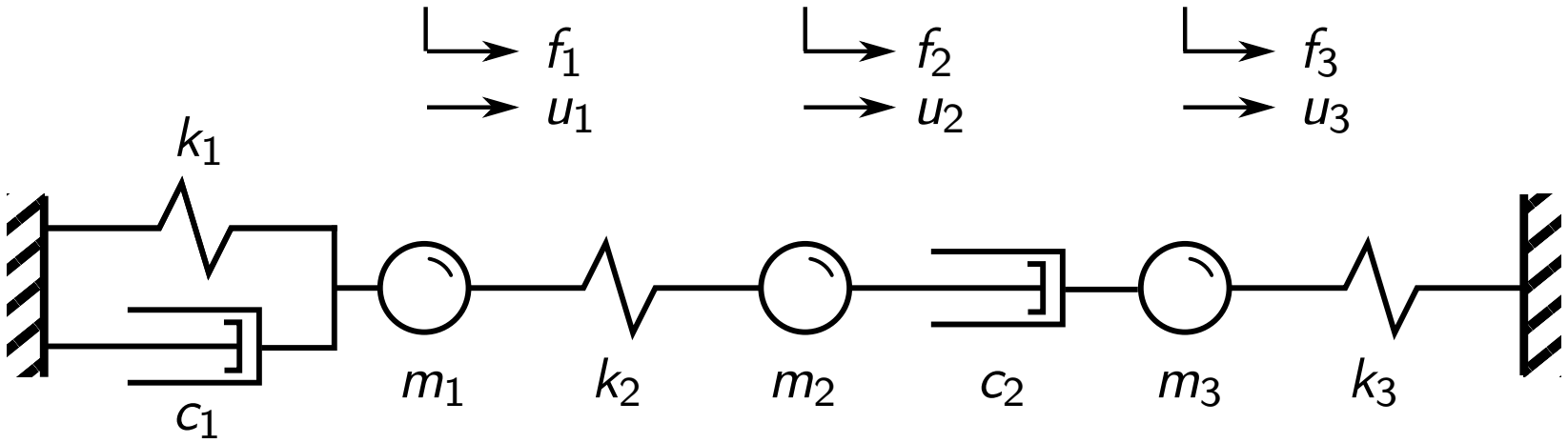
Dynamic systems with concentrated parameters
Excercises

Simulation Methods in Acoustics
03.10.2017.

Dynamic system matrices



$$k_1 = 1\,000, \quad k_2 = 2\,000, \quad k_3 = 1\,000 \quad [\mathrm{N/m}]$$
$$c_1 = 10, \quad c_2 = 20 \quad [\mathrm{Ns/m}]$$
$$m_1 = 1, \quad m_2 = 2, \quad m_3 = 2 \quad [\mathrm{kg}]$$

- Assemble the stiffness, damping and mass matrices of the system shown above.

$$\mathbf{Ku} + \mathbf{Cv} + \mathbf{Ma} = \mathbf{f} \tag{1}$$

- *Hint: it is easier to write the whole matrices at once than assembling element-by-element in this case*

Time Domain Solution – State Space Model

- Assemble the state space model of the system in the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}, \qquad \mathbf{x} = \begin{Bmatrix} \mathbf{v} \\ \mathbf{u} \end{Bmatrix} \tag{2}$$

- Display the analytical free displacement ($\mathbf{f}(t) = \mathbf{0}$) response $\mathbf{u}(t)$ of the initial value problem

$$\mathbf{u}(t=0) = \begin{Bmatrix} 1 & 0 & 0 \end{Bmatrix}^{\mathrm{T}}, \qquad \mathbf{v}(t=0) \ = \begin{Bmatrix} 0 & 0 & 0 \end{Bmatrix}^{\mathrm{T}} \tag{3}$$

- *Hint: use the Matlab function* `expm(A*t)` *or the spectral decomposition* `[V,D]=eig(A)` *for matrix exponential*
- Compute the free response using the forward Euler scheme.
  - Select an appropriate time step h in the stability region.
  - Compare the analytical and numerical solutions.
    Does the result match your expectations?
  - Select another h outside the stability region. Check the result.

Normal modes

- Generate a bridge model using the command

```
[nodes, elems] = create_bridge(20, 4);
```

- Generate the mass and stiffness matrices of the bridge as

```
EA = 1e7; rhoA = .1;
[K, M] = truss_stiffness(nodes, elems, EA, rhoA);
```

- Modify the mass and stiffness matrices of the model so that the lower corner DOF of the bridge are clamped ($\mathbf{u}_c = \mathbf{0}$).
- Compute the normal modes of the constrained system

```
[Psi, Lambda] = eigs(K, chol(M), nModes, 'sm',...
            struct('cholB', 1, 'symm', 1) );
```

- Display the modes of the system (function: `plot_truss`)

Modal Solution

- Compute the generalized system matrices and the generalized forces assuming constant vertical loading force on the top nodes.
- Compute and display the generalized displacements in the frequency range $0 < f < 50\ \mathrm{Hz}$.
  Select the modal base by applying Rubin's rule.
- Display the spectrum of the horizontal and vertical displacement of a freely selected node.
- Apply proportional damping to the system so that the damping coefficient is $\xi = 1\%$ at the lowest eigenfrequency, and increases with frequency.