Finite dimensional approximation of functions
Excercises

Simulation Methods in Acoustics
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A quick introduction to Matlab's symbolic toolbox

- Using the Symbolic Math toolbox, Matlab can handle expressions in a symbolic level. This means that functions can be defineed and manipulated without substituting any fix values as their parameters. Instead, symbols can be used.

1. Familiarize yourself with Matlab's symbolic capabilities by examining the script `symbolic_intro.m`

Polynomial approximation of the normal distribution

- Compute the N-th order polynomial approximation of the standard normal PDF[1] $f(x) = \mathrm{e}^{-x^2/2}$ over the interval $-3 \leq x \leq 3$. *(Hint: include the constant polynomial too.)*

1. Collocation method
   - Apply the collocation method with N equidistant collocation points inside the interval
   - Display the approximation error
   - Compute the L2 norm of the approximation error

$$\|r(x)\|_{\mathrm{L2}} = \int_{\Omega} r^2(x)\,\mathrm{d}x$$

2. Galerkin method
   - Apply the Galerkin method to compute the approximation
   - Compute the L2 norm of the approximation error and compare it to the error of the collocation method

[1] Note that the constant scaling factor $1/\sqrt{2\pi}$ is omitted.

Solving the Poission equation I.

- In this example we will solve a BVP of the Poisson equation, i.e. $-\Delta u(x) = f(x)$ in one dimension
- The problem is defined as:

$$\begin{aligned} \text{PDE}: \quad & -\frac{\partial^2 u(x)}{\partial x^2} = f(x) = 4\left(x - \frac{1}{2}\right)^3 \quad x \in \Omega = [0,1] \\ \text{BCs}: \quad & u(0) = 0 && \text{(Dirichlet)} \\ & u'(1) = 0 && \text{(Neumann)} \end{aligned}$$

1. Is the problem expected to be well defined? (Check if BCs are given for the whole boundary.)
2. What do you expect of the solution $u(x)$? *(Hint: $u''(x)$ is $f(x)$, a polynomial of third order in this case.)*
3. Recall the steps of defining the weak form of a BVP and construct the weak form the above BVP!

Solving the Poission equation II.

4. Choose a finite dimensional polynomial function space $\mathcal{U}$ to represent the solution function![2] $\mathcal{U} = \{x, x^2, \ldots, x^N\}$
5. Use the Galerkin method[3] and calculate the matrices and the r.h.s. vector of the BVP. *(Hint: try to be as generic as possible, e.g. N should be a variable of your program.)*
6. Solve the problem with $N = 3$, 4, 5, and 7. Display the results and compare $-u''(x)$ with $f(x)$. Explain the results! *(Hint: compare the coefficients for $N = 5$ and $N = 7$.)*

** Solve the same problem with the Dirichlet B.C. modified as $u(0) = 1/2$. First, use the same function space $\mathcal{U}$ as above! Explain the result! Extend the function space and impose constraints (use the Lagrange method e.g.) to arrive at correct results.

[2] This is the *trial function space*.

[3] i.e., let the test function space be same as the trial space.